Central Connecticut State University
Department of Engineering
Mechanical Engineering Program

ME 498 Senior Design Project Report

Fall 2018

Formula SAE Rear Suspension Design, Optimization, Analysis, & Fabrication Final Report

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This report is written in partial fulfillment of the requirements in ME 498 – Senior Design Project. The contents represent the opinion of the authors and not the Department of Engineering

Abstract

The 2018 CCSU FSAE Team needed a rear suspension system to be completed utilizing previous suspension and component geometry. Research on race car dynamics gave background to all design components and engineering standards. Bell cranks, push rods, and mounting was developed within the design space to achieve an optimal rear suspension for the Formula Race Vehicle. New layouts of the shock assemblies allow for ideal accessibility and fine tuning of the suspension while eliminating all interferences seen by previous designs. Topology optimization was utilized to introduce a new design tool, as well as, reduce overall weight in the vehicle using best stiffness to weight goals for design. Goals for weight reduction, cost minimization, and manufacturing labor time were all exceeded. Weight was reduced by 31.5% using new layouts, topology optimization, and material changes. Costs were condensed to $54 of the $250 budget by utilizing previous materials and components. Labor was reduced by 28% using CNC machining and good manufacturing practices and processes. Design of the system includes in-plane bell crank motion and a Motion Ratio of 2 ensuring optimal performance of the system. Applying Finite Element Analysis concluded a system that can withstand all maximum forces and stress applied by a safety factor of at least 2.

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Nomenclature

A-arms: Component of the suspension system that allows the wheel to translate vertically

Altair University: Computer Aided Engineering Software

ANSYS: Engineering Simulation software

Axel: Component of a vehicle that transmits power from the drive train to the wheel

Bell Crank: Lever that translate motion from push or pull rod to the shock assembly

Bridgeport: Milling Machine brand

CAD: Computer Aided Drafting

CCSU: Central Connecticut State University

CNC: Computer Numerical Control

Contact Points: Connections between sprung and unsprung masses

Control Arms: Another name for the A-arms. Control the vertical motion of the wheel

Damping Ratio: Measure describing how oscillations decay after a disturbance

Differential: A set of gears that allow a vehicles wheels to move at different speeds when cornering

Dynamic: The scientific study of motion and what causes or stops motion

FEA: Finite Element Analysis

FSAE: Formula Society of Automotive Engineers

Fulcrum: The rotation point on a bell crank or lever

Hertz: Measurement of frequency

HSMWorks: Software within D'Assaults SolidWorks that allows a user to write G code for a CAD Model

In: British measurement of length

Inspire:

Instant Center: Imaginary point at which the upper and lower A-arms pivot

Kinematics: the branch of mechanics concerned with the motion of objects without reference to the forces that cause the motion.

Lateral: Direction to the side

Lathe: a machine for shaping wood, metal, or other material by means of a rotating drive that turns the piece being worked on against changeable cutting tools.

Lb: Unit of weight

Lb/in: Measurement unit of linear density

Lbf: English Measurement of force

Longitudinal: Lengthwise direction rather than across or side to side

Mesh: Interlaced network structure

Mesh Convergence: Term used in FEA when the nodal network is studied to find the point where the mesh converges to a central value that will yield the same result in the stress

Motion Ratio (M.R.): Ratio between amount of wheel travel to amount of shock travel

Plasma Router: CNC router machine used to cut out components from stock material using plasma

Psi: Pounds per square inch, measurement of pressure

Pull Rod: Component in a suspension that runs from the upper control arm to the bell crank that translates vertical motion of the wheel to the bell crank

Push Rod: Component in a suspension that runs from the lower control arm to the bell crank that translates vertical motion of the wheel to the bell crank

Ride Frequency: Undamped natural frequency of the body in ride

SAE: Society of Automotive Engineers

Shock Assembly: Assembly of the suspension consisting of a spring that absorbs the forces acting on the suspension

SLA: Short Long Arm

SolidWorks: Solid modeling computer aided design and computer aided engineering computer program

Spring Rate: Amount of weight needed to move a spring one inch

Sprung Mass: Portion of the vehicle weight that is supported by the suspension system

Static: Lacking in action, movement, or change

Struts: Structural component that resists longitudinal compression or tension

Suspension System: tires, tire air, springs, shock absorbers and linkages that connects a vehicle to its wheels and allows relative motion between the two

Tie Rods: Component of the suspension system that prevents the wheels from rotating side to side

Topology Optimization: mathematical method that optimizes material layout within a given design space, for a given set of loads, boundary conditions and constraints with the goal of maximizing the performance of the system

Unsprung Mass: Weight of the suspension system itself and anything connecting to the wheel

Upright: Component of the suspension system that connects all other components to the wheel

Vertical: Perpendicular direction to the horizontal or base plane

Acknowledgment

I would like to take this opportunity to thank the many people that have helped me with this study and report. First, my advisor, Dr. Moore, who guided me through rough patches in my design and analysis, while also introducing me to new software and capabilities that expanded my horizons in design. I would also like to thank my co-workers at The Lee Company, who have taught me to how to think outside of the box.

I. Introduction

Formula SAE is a competition that allows collegiate students to use their engineering knowledge and creativity to design, analyze, manufacture, and race a formula one style race car. This competition brings the students far beyond the classroom by applying their skills to a real-world application. This allows engineering students to expand their team building, project management, and communication dexterity while under design constraints that will surely prepare them for their future careers.

Each teams' vehicle will be subject to both static and dynamic testing and granted points based on the performance. The team with the most overall points will be awarded the winner. Judging events include cost analysis, inspection, individual performance trials, and high-performance track endurance. The point system of each even is as follows:

Static Events:			
	Presentation	75	
	Engineering Design	150	(may be changed to 200 for 2018)
	Cost Analysis	100	
Dynamic Events			
	Acceleration	100	
	Skid-Pad	75	
	Autocross	125	
	Efficiency	100	
	Endurance	275	(may be changed to 225 for 2018)
Total Points		1,000	

Figure 10: FSAE Event Point System

The suspension is an integral system of any car that comprises of components including the push/pull rods, bell cranks, struts, A-arms, tie rods, uprights, and wheels. Each mechanism serves an important role in comfort and performance of the vehicle. The suspension system also includes the sprung and unsprung masses that are subject to many different forces, whether from driving surface imperfections, braking, accelerating, or cornering.

One of the main purposes of the suspension system is to keep the wheels in contact with the road and either maintain or change the geometry as the vehicle is exposed to different classes of forces to optimize performance. When designing for the complex geometry, the vehicle dynamics is crucial. The first step in vehicle dynamic design is to identify the axis and the sprung and unsprung mass being considered. The sprung mass is the portion of the vehicle weight that is supported by the suspension system. Conversely, the unsprung mass is the weight of the suspension system itself and anything connecting to the wheel. The unsprung mass, sprung mass, and vehicle axis can be seen in the figures below.

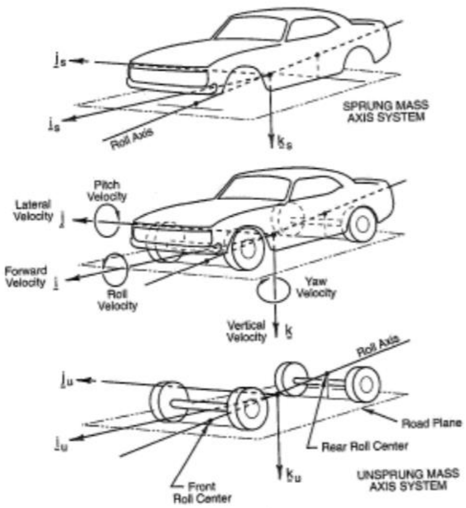


Figure 11: Sprung and Unsprung Mass Axes

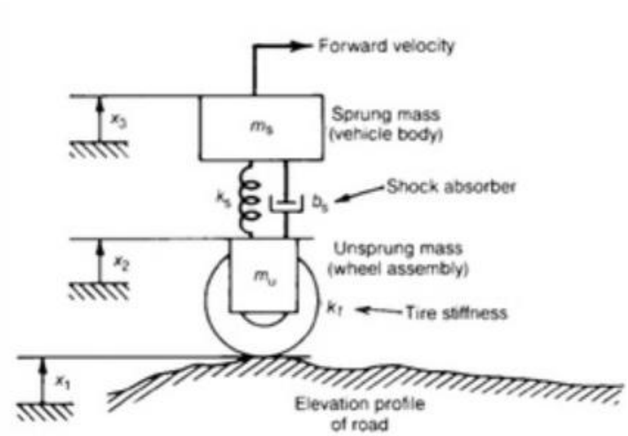


Figure 3: Sprung and Unsprung Mass Representation

There are many design variations of suspensions, but they all include contact points, which can be defined as the connections between the sprung and unsprung masses. The relative motion of the vehicle is prescribed by these connections and every force scenario that the vehicle experiences acts through them, so each component should be designed carefully and modified to accept and absorb the forces appropriately.

Suspension systems are designed to equip a multitude of vehicles. There is a diverse class of vehicles ranging from race cars such as the formula one, commuter cars, and luxury vehicles. Each class will accommodate either performance, comfort, or a mixture of both. To accomplish this, the suspension must be compromising and have trade-offs. For example, wheel grip to the driving surface may have to be compromised if lower center of gravities or vehicle heights are desired. The accommodations can be classified by ride frequencies, which is determined mostly by the stiffness of the spring used in the system and can be defined by the undamped natural frequency of the car during operation. Ride frequencies are considered high when the spring stiffness is high, resulting in minimal travel of the sprung mass. With high ride frequencies comes benefits in low ride height, resulting in a low center of gravity. Low ride frequencies allow more travel in the sprung mass, due to lower stiffness in the strut, introducing more wheel grip. Ride frequencies are measured in Hertz and range from about 0.5-1.5 Hz for commuter and luxury vehicles, 1.5-2.0 Hz for race vehicles, and 3.0-5.0 Hz for high downforce race vehicles. [8]. The ride frequency can be calculated using the equation:

$$f = \frac{1}{2\pi} * \sqrt{K/M}$$

Where f is measured in Hertz, K is the spring rate, and M is the mass. To find the desired spring rate for a chosen ride frequency, apply the equation:

$$K_s = 4\pi^2 f^2 m_s (M.R.)^2$$

Where m_s is the sprung mass and M.R. is the motion ratio found by $M.R. = \frac{Wheel\ Travel}{Spring\ Travel}$.

To expand, commuter cars are designed based on lower ride frequencies because the design goal is to make a vehicle that is efficient, handles well, and absorbs bounce and jounce of the rough roads that it may experience since these cars will be driven quite often. On the other hand, race vehicles desire a higher ride frequency due to the advantages of low height and center of gravity, giving the car better aerodynamics. Comfort in these cases are compromised for vehicle performance. Typical race tracks are very smooth and may not need to accommodate for large displacements due to poor road surface conditions.

Independent suspension systems of the front and rear may not be designed to perform the same in the ride frequency department. On a track, if there is a disruption in the front suspension, there will be a delay in the rear until it experiences the equivalent disruption. This will throw the frequencies of the front and rear suspension out of phase. Tuning the front and rear suspensions is critical in this way. Race cars are typically set up with a higher ride frequency in the front than the rear. This is desirable because a lower ride frequency in the rear allow the vehicle to achieve more grip between the tire and the road and allow more speed out of the exit of a corner. The rear tires desire more grip because they are being driven by the engine. With a high ride frequency in the front, the vehicle has improved maneuverability by obtaining a faster transient response at corner entry. [8].

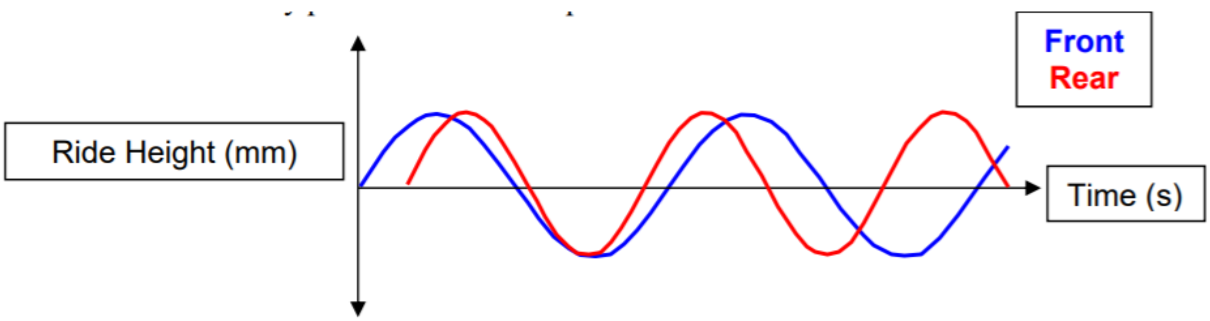


Figure 4: Ride Frequency of Front and Rear

Damping Ratios in the strut, allows for quick responses and less overshoot. Race cars desire to have a high damping ratio overall which will aid in achieving increased vehicle control. As stated with the low ride frequencies, the rear suspension desires a lower damping ratio than the front to maintain the mechanical grip of the tire with the road surface. Typical damping ratios for race cars range from 0.65 to 0.7. Our suspension will be designed to include the 0.7 damping ratio for added benefits.

A major component of an independent suspension system is the control arms. There are many different control arm designs, but they all contain five linkages to account for the five degrees of freedom of the system. Any body in space has six degrees of freedom, but the job of the control arms is to control the motion of the wheel relative to the car body in one dictated path, which eliminates one degree of freedom. Previously designed for the 2018 rear suspension is the A-arms that are set up in Short Long Arm (SLA) double wishbone format. Five linkages can be counted; one in the toe link, two in the upper A-arm, and two in the lower A-arm. The SLA double wishbone format allow for many advantages and is among the most popular of suspensions due to its versatility in performance without much compromise.

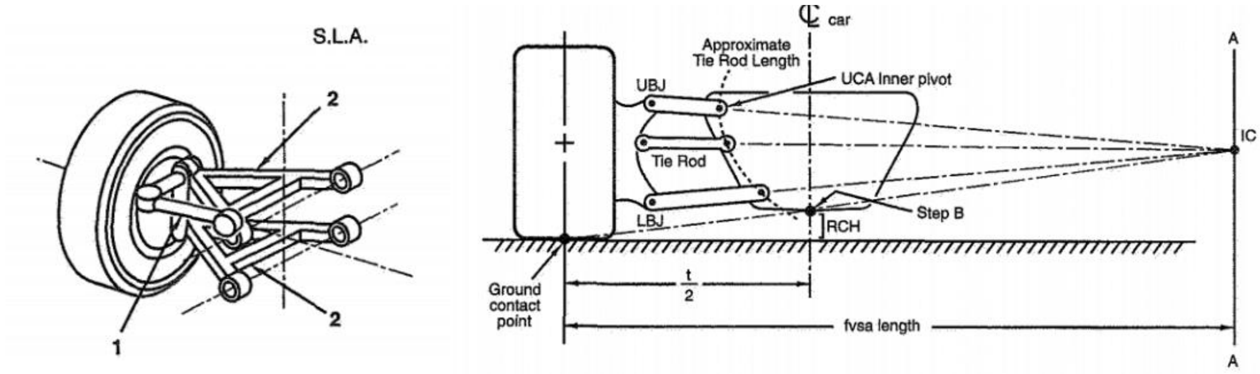


Figure 5: SLA Double Wishbone Suspension

The geometry and kinematics of the design are focused on the instant center defined as the imaginary point at which the upper and lower A-arms pivot. The vehicle is viewed in three dimensions, but viewing the problem at hand as two dimensions allows many of the suspension properties to be defined. A front view projection of the instant center describes the camber rate change, scrub motion, and steering characteristics. Rotating the view 90 degrees and projecting the instant center will describe the anti-lift and anti-squat parameters and caster rate change. [1]. With the SLA double wishbone suspension previously designed, design and optimization will be completed on the push rod, bell crank, and strut.

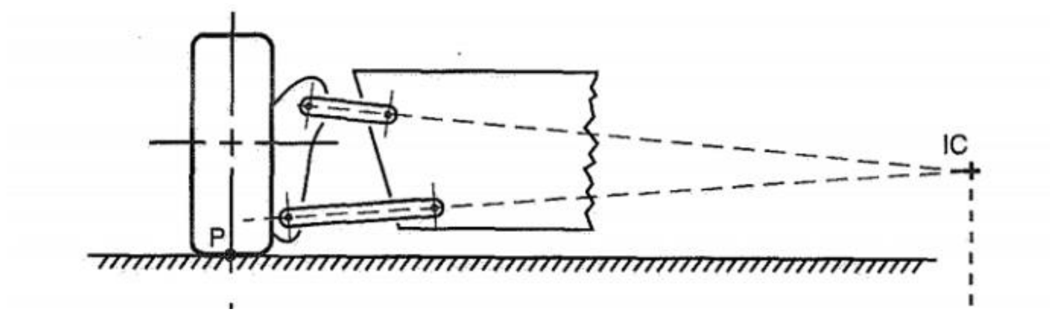


Figure 6: Instant Center

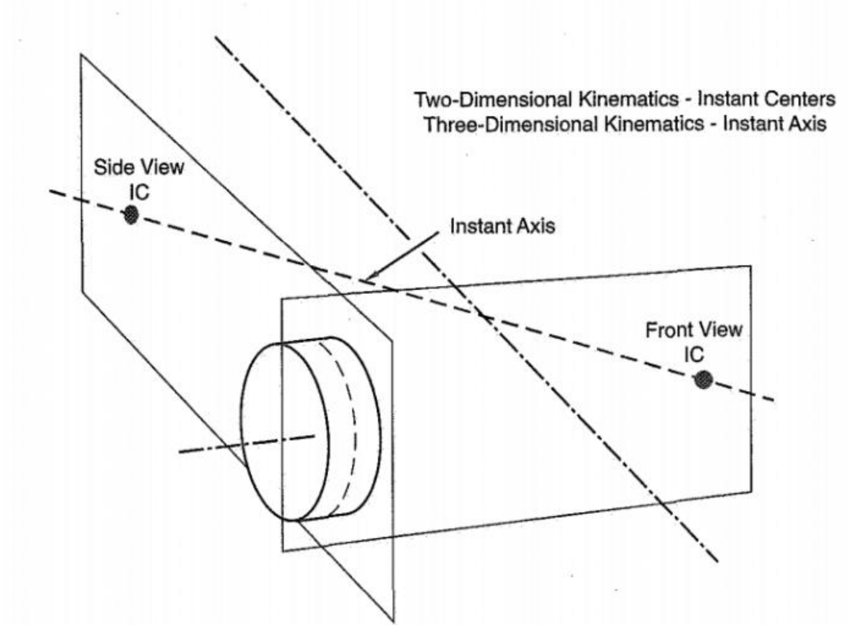


Figure 7: Instant Center and Instant Axis

The bell crank, sometimes referred to as a rocker arm, is a lever that is attached to the pushrod, the frame, and the shock assembly. The purpose of the bell crank is to translate motion from the wheel and pushrod to the shock assembly by rotating the bell crank around a fulcrum. Keeping the bell crank in plane with the translation is ideal so that the bell crank is only translating the motion in the planes of the shock and pushrod. Translating in any other planes with compromise full advantage of the bell crank. The geometry of the bell crank can vary based upon desired shock and wheel travel. The degree of motion can be classified by the Motion Ratio, which is defined as:

$$M.R. = \frac{Wheel\ Travel}{Shock\ Travel}$$

The dimensions of the bell crank arms are equal to the moment arms and can be used in the equation as well by:

$$M.R. = \frac{L_1}{L_2}$$

Where L_1 is the length from the fulcrum to the push rod joint and L_2 is the distance from the fulcrum to the shock joint. When determining the appropriate motion ratio, knowledge of the shock travel and desired lateral wheel translation is needed. A common mistake when determining the motion ratio is assuming that the allowable travel is the full length of the shock, when in reality you must know the length of compression of the shock when under the load of the sprung mass. Only then can you determine the amount of shock travel left and design within those constraints.

Rear Wheel Motion & Spring Information			
Item	Symbol	Value	Units
Vertical Wheel Displacement - Bump	δ_B	1.75	in
Rear Sprung Corner Weight	$W_{s,corner}$	148.35	lb
Rear Unsprung Corner Weight	$W_{u,corner}$	11.15	lb
Bell Crank Pivot to Pushrod Length	L_1	2	in
Bell Crank Pivot to Shock Length	L_2	1	in
Bell Crank Pivot to Anti-Roll Bar Length	L_3	4	in
Uncompressed Spring Length	$L_{s,u}$	4.75	in
Maximum Shock Travel	$\Delta L_{S,MAX}$	2.25	in

Table 1: Rear Wheel Motion and Spring Information

Rear Spring Length & Motion Calculations			
Item	Symbol	Value	Units
Initial Spring Compression	$\Delta S_{initial}$	1.58	in
Total Wheel Movement	δ_{MAX}	3.33	in
Motion Ratio	**MR**	**2.00**	**-**
Total Spring Movement	ΔS_{MAX}	**1.66**	**in**
Required Spring Rate	K_s	**376.25**	**lb/in**
Ratio - Uncompressed Spring Length to Total Spring Movement*	$R_{SL/SM}$	**2.86**	**-**

Table 2: Rear Spring Length and Motion Calculations

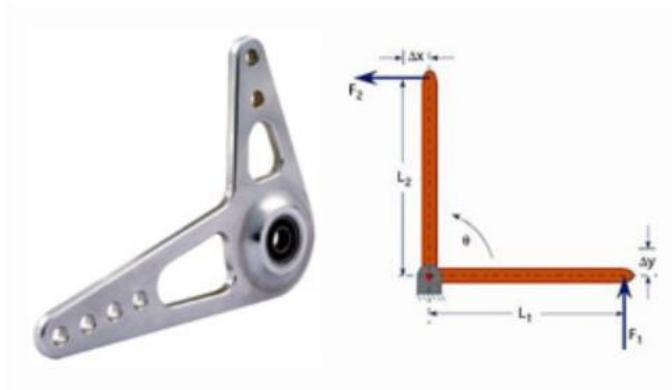


Figure 8: Bell Crank

Push and pull rods are used in the suspension system to transfer the forces of the wheel to the bell crank and finally to the shock assembly. The pull rod is typically used in a lower nose vehicle. The connection point of the pull rod on a double wishbone suspension system is on the upper A-arm to the bell crank and shock assembly. The capability to package the suspension components lower, allows the center of gravity to be lowered and increase the aerodynamics. Conversely, push rods connect to the lower A-arm and the suspension components are assembled high on the frame. The choice between the two comes down to center of gravity and the geometry package of the vehicle. Push rods are more easily accessible, while pull rods are difficult to access for possible tuning.

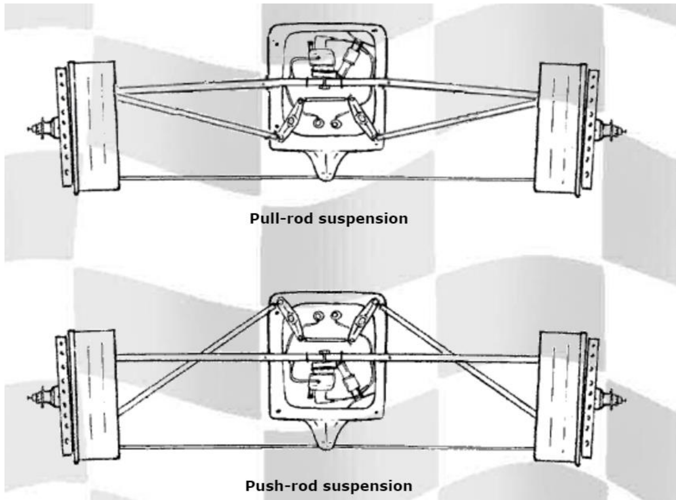


Figure 9: Push and Pull Rod Systems

Previous work in the Spring of 2017 on the rear suspension utilized a push rod, plated bell cranks, and SLA double wishbone A-arm set up. In the fall of 2017, the FSAE team optimized the suspension set up utilizing the SLA double wishbone A-arm system again. The new suspension design did not include the push rod, bell cranks, or shock assembly, which needs to be completed for localization of the hard points to the chassis. Spring 2017 rear suspension's Motion Ratio equated to 1.67, while keeping the bell crank motion in plane. Total weight of the rear suspension system totaled to 10.65 lbs. The budget for the team was $1000 and they were able to come in under budget with a total of only $202.45. Manufacturing components of the suspension amounted to 12 hours of production time.

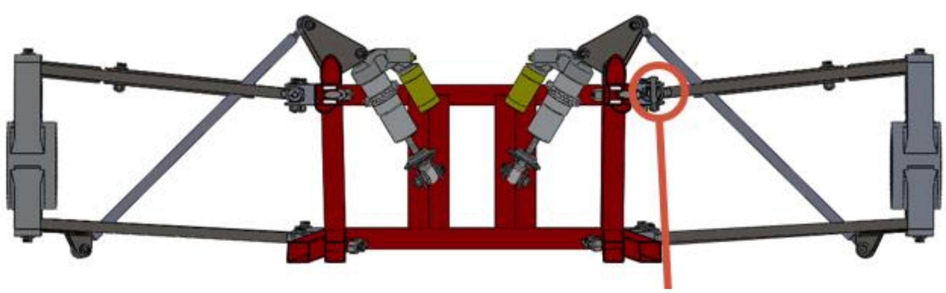


Figure 10: Previous Design Rear View

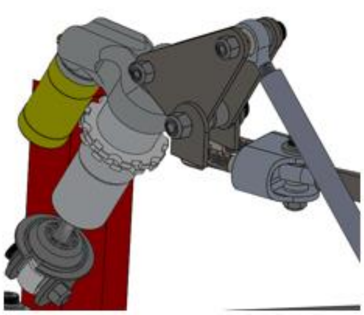


Figure 11: Previous Design Close Up

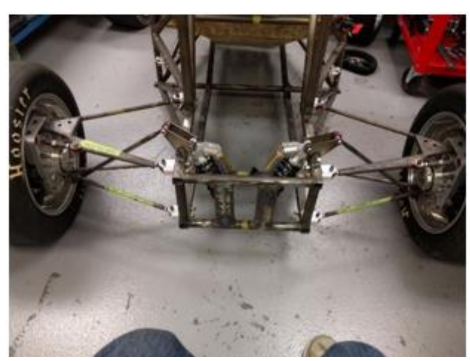


Figure 12: Previous Design Assembled

The Spring 2017 team also developed an intuitive excel calculator capable of calculating input loads, performance characteristics, and component stress elements. Necessary equations for applicable loads, performance, and stresses will be confirmed for each component.

Topology Optimization is a new tool to the available engineering software such as ANSYS, SolidWorks, Inspire, and Altair University to name a few. Many times, a designer is faced with challenges of a redesign of a component or an unusual design space. These design challenges are made even more difficult when the deflections are to be held minimal within the space or the weight is an issue. Topology optimization can tackle these problems, making engineers jobs easier.

Topology optimization is capable of finding the best distribution of material possible given optimization goals and design constraints. Common optimization goals include maintaining stiffness while reducing weight and minimizing weight while sustaining deflection limits. Constraints that can be applied include symmetry, constant thickness, preserved regions, and applicable load cases. Symmetry and thickness are important constraints for manufacturing, since fabrication has its own limitations. Preserved regions are essential to outline where you do not want any material to change, such as areas where you will have a pin or fastener. Load cases can be applied just as in Finite Element Analysis. Capabilities of multiple load cases allows the designer to have a more accurate optimization.

Behind the scenes, the software is maximizing stiffness or minimizing compliance. The software starts with calculating the stiffness matrix. From your desired weight, an inequality constraint on mass is applied.

$$\sum_{e=1}^{N} \{v_e\}^T \rho_e \leq M_{target}$$

The optimization will have a nonlinear penalty function on density. The point of this is to aid in manufacturing. Using a linear penalty function, thin sections of the material will survive. On the other hand, with the non-linear penalty function, as the density approaches smaller values, the density becomes negligible.

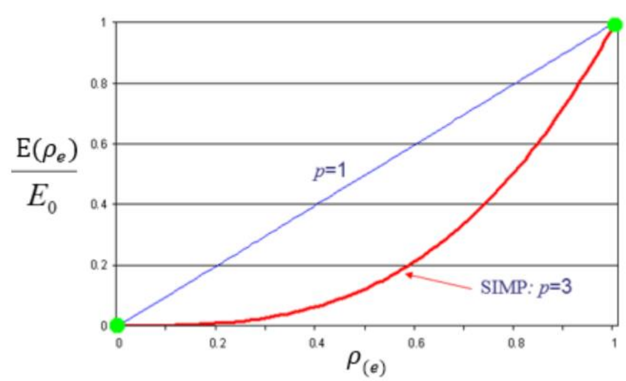


Figure 112: Topology Optimization Penalty Function on Density

$$\frac{dC}{d\rho_e} = -p(\rho_e)^{p-1}[u_e]^T[K_e][u_e]$$

The software will take the gradient of the calculated stiffness with respect to density, which produces a slope. The slope is then ranked from higher slopes to lower slopes. Each iteration of the optimization will use a step function to reduce the stiffness. The higher the slope, the smaller the impact on the density as the step function is applied. Conversely, the smaller the slope, the more impact on the density. Because of the non-linear penalty function, the stiffness steps of the smaller slopes are increased exponentially, which makes the density negligible. These sections are then removed from the component. [7]

II. Statement of Work

A. Problem Definition

The CCSU FSAE Rear Suspension Team will redesign and optimize Spring 2017 rear suspension, while packaging the suspension components within the Fall 2017 SLA Double Wishbone system. Relocation of the struts is the first priority due to the interference with the new differential being implemented. A 30% weight reduction of the utilized suspension components is also desired while maintaining structural integrity. Manufacturing time and costs are desired to be reduced by 15%. Utilizing previous componentry is ideal as it leads to time and cost reductions.

B. Engineering Process

To start the project, a problem needed to be defined. Contacting the advisor and president of the FSAE Club allowed for a plethora of projects to make the race car complete. One of the complications was the rear suspension. The Fall 2017 team designed the SLA Double Wishbone suspension but left future work of the push/pull rod system, bell crank, and strut to be completed. Since the chassis is being made at the end, mounting points for the suspension also needed outlining.

Research was next on the agenda after defining our problem. Using race car and dynamic text books, I was able to find a lot of information about how suspension systems are designed for race vehicles along with the engineering and math behind the scenes. The J-store at CCSU provided various articles and papers on rear suspensions that ended up being crucial to understanding of the system, as well as different design ideas. Research online also contributed to possible design ideas from previous FSAE team vehicles.

After finding many design options, research on the rules and regulations for FSAE was conducted to understand any specifications for the rear suspension. Research of the SLA Double Wishbone set-up was also expanded, especially the exact fabricated parts to have knowledge of the geometric envelope for further completion and development of the system.

Brian-storming and evaluation of the designs that met the requirements and specifications was conducted. Design matrices were developed to decide between the push or pull rod system, as well as the bell crank make up (billeted or plated). The design matrices allowed for the decision of the plated bell crank and push rod system.

Using CAD software, large assemblies of the suspensions were used to layout the componentry of the system and draw the parts that the system comprises of. SolidWorks was the main program used for the 3D modeling each prototyped part.

After the layout of the system and modeled prototypes were established, topology optimization was researched and learned to further reduce weight in the bell crank. After topology optimization, finite element analysis was conducted for the given load cases and load angles on each component of the system to determine its strength and reliability. Yielding a Factor of Safety above 2 was an underlying goal.

Once the FEA analysis was validated, the parts were fabricated. The bell crank was manufactured on a CNC mill with a program written using HSMWorks software. The shock mounts were cut on the plasma router for ease and new learning experiences. Lastly, the fulcrum pin was turned on the manual lathe. The components will be assembled when the chassis is complete.

C. Design Constraints

A design constraint is any restriction that defines a project's limitations. The three most significant project constraints are schedule, cost and scope. The time frame for this project was 45 weeks. The first half consisted of the research, development, scheduling, and costing, while the second half comprised of the majority of the design, analysis, and manufacturing. Gantt charts were used to layout expected completion dates of different phases of the projects.

	Task	Assigned To	Start	End	Dur	%
	Project		1/23/18	12/1/18	218	
1	Determine FSAE subsystem to redesign		1/23/18	1/31/18	6.979	
2	Form Group		1/23/18	2/9/18	14	
3	Review Previous Projects and Work		2/9/18	4/24/18	51.979	
4	Download Software		2/9/18	3/9/18	20	
5	Research Suspension Design		2/9/18	5/8/18	62	
6	Document Literature Review		2/9/18	5/8/18	62	
7	Specification Sheet		3/24/18	4/3/18	7	
8	Weekly Project Status Update (ME 497)		3/27/18	5/15/18	36	
9	School Paperwork		3/27/18	4/4/18	7	
10	Gantt Chart		3/28/18	4/10/18	10	
11	Written Proposal and Presentation (ME 497)		4/5/18	5/8/18	24	
12	Determine all Possible Rear Suspension Solutions		4/10/18	4/24/18	11	
13	Prototype		4/16/18	4/24/18	7	
14	Learn Optimization Software		8/27/18	9/10/18	10	
15	Perform Iterations and Re-analyze Suspension		9/10/18	10/1/18	16	
16	Create Solidworks Models of Rear Suspension		10/1/18	10/10/18	7	
17	Optimize Bell Crank		10/2/18	10/10/18	6	
18	Created Models of each Suspension Component		10/8/18	10/15/18	5	
19	Analyze Components		10/15/18	10/25/18	9	
20	Create Solidworks Drawings of Components for Manufacturing		10/16/18	10/18/18	3	
21	Learn CAM tools in Solidworks		10/18/18	10/19/18	2	
22	Manufacture Components		10/19/18	11/11/18	16	
23	Written Proposal and Presentation		11/12/18	12/1/18	14	
24	Test Components		11/12/18	11/20/18	7	
25	Re-Design and Fabricate as Needed		11/20/18	11/25/18	3	

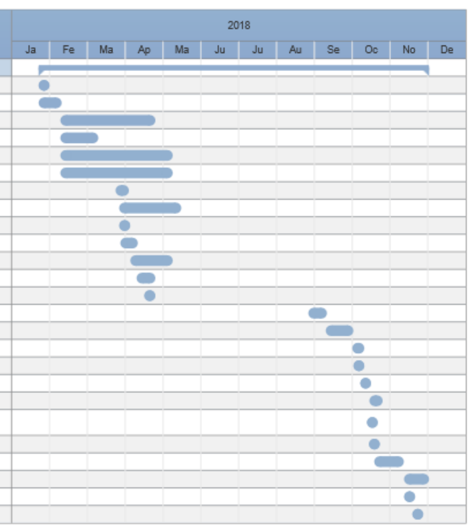


Figure 113: Gantt Chart

2018-2019 FSAE Rules and Regulations	
Constraint	**Value**
Useable Wheel Travel	2 inches
Minimum Wheel Rebound	1 inch
Minimum Wheel Jounce	1 Inch
Mounting Points	Visible to Technical Inspectors
Ride Height	No Permissible Ground Contact
Fasteners	Meet or Exceed SAE Grade 5 or Equivalent
Securing of Fasteners	Positive Locking Mechanism

Table 3: FSAE Rules and Regulations

Another larger constraint was the FSAE Rules and Regulations that lay out all of the requirements of each system in the vehicle. For the rear suspension, the constraints are described in the table above. FSAE rules require a minimum of two inches of useable wheel travel for a sufficient system. In other words, the suspension must allow at least one inch of rebound and one inch of jounce due to bumps in the road surface. The vehicle may not contact the ground, so initial ride height is important to consider in the travel of the wheel. For the initial inspection, all mounting points must be visible or made visible to the technical inspectors. Each fastener must also be positively locking to ensure the system stays assembled and the fasteners must meet or exceed SAE grade 5.

For my design portion of the rear suspension, I was also bound by the previously designed uprights, A-arms, tie rod, axle to the differential, and the differential itself. The Fall 2017 suspension team chose to use the SLA Double Wishbone set up as seen in the figure below. Designing within the SLA Double Wishbone space was achieved by the Spring 2018 Team, but interferences were found at the left shock assembly and the new differential. Therefore, these interferences needed to be eliminated while complying with all previously stated constraints.

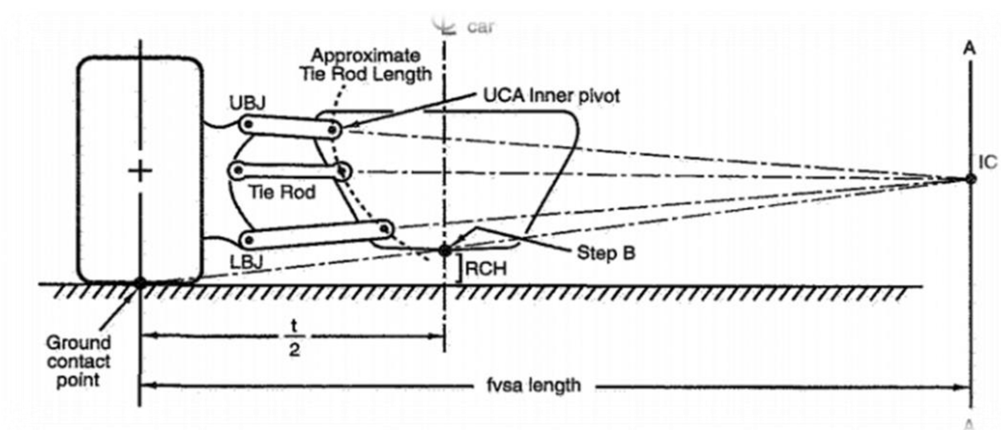


Figure 114: SLA Double Wishbone Suspension Set Up

D. Alternative design ideas and methods

Laying out the system yielded many design possibilities. Depending on the positioning of the shock assemblies, the design of the bell crank, push/pull rods, and mounts all changed dramatically. There are two types of bell cranks to consider, a plated or billeted. Each has its positives and negatives and choices were changed by each of the layouts. The geometry of the bell crank is also constantly changing depending on the Motion Ratio that is desired and required for the system. Push and pull rods are used in many vehicles and depending on the layout of the shock assemblies, the choice for either changes because of the geometry space and accessibility. Below are figures of a few design layouts that were in contention for being chosen with distinct differences of the push/pull rods, bell cranks, and the mounting.

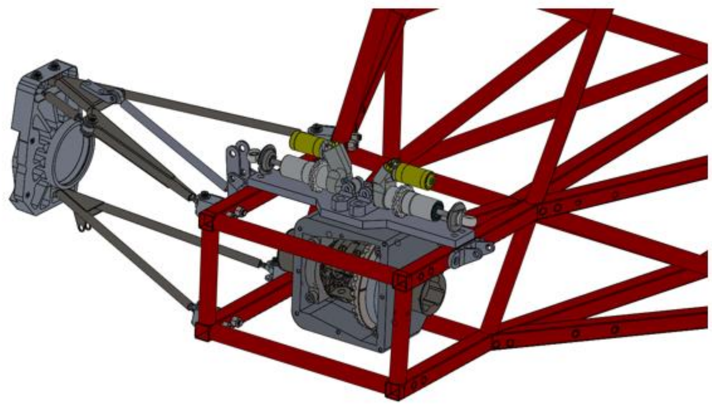


Figure 115: Shock Assembly above Differential

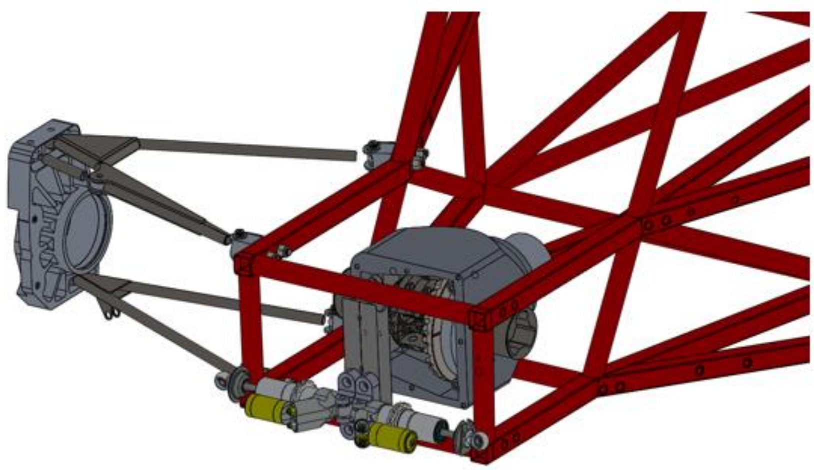


Figure 116: Shock Assembly in Rear

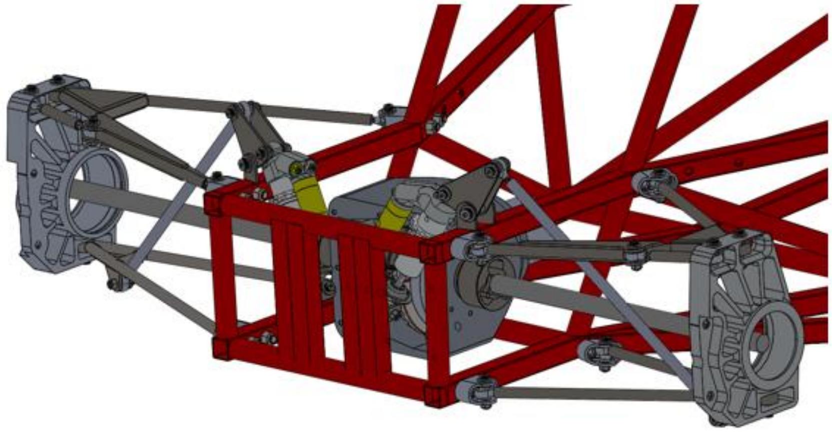


Figure 117: Shock Assembly Protected in Rear

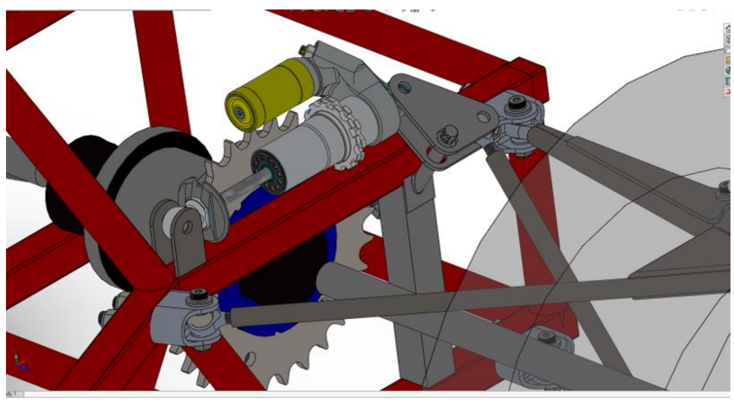


Figure 118: Shock Assembly Horizontal

III. Results

A. Selection of best design idea

One of the first decisions to make was for the type of bell crank that I wanted to utilize in my design. The two types include a billeted and a plated design as seen below. Each has its benefits and drawbacks, so a design matrix was used to decide between the two based on a handful of criteria important to the vehicle's compliance and performance. As seen below, a plated bell crank was chosen, winning out to the billeted bell crank from weight and manufacturability.

Figure 20: Billeted Bell Crank

Figure 21: Plated Bell Crank

Bell Crank				
			Options	
			Plated	Billeted
Criteria	**Weight**	**Multiply**	**Score**	**Score**
Strength	100	x	8	10
Manufacturability	80	x	10	7
Weight	90	x	9	8
Cost	80	x	3	2
Assembly	60	x	5	7
			Total	
			2950	2860

Figure 22: Bell Crank Design Matrix

The next design consideration was for the push or pull rod component. Again, each type of rod has its benefits and drawbacks. The criteria exceeds the amount for the bell crank since there is more to consider for compliance and constraints as described earlier. A design matrix was used to determine a push rod as the best type of rod to use in my system. The drivers of the push rod include the accessibility for tuning and overall geometry.

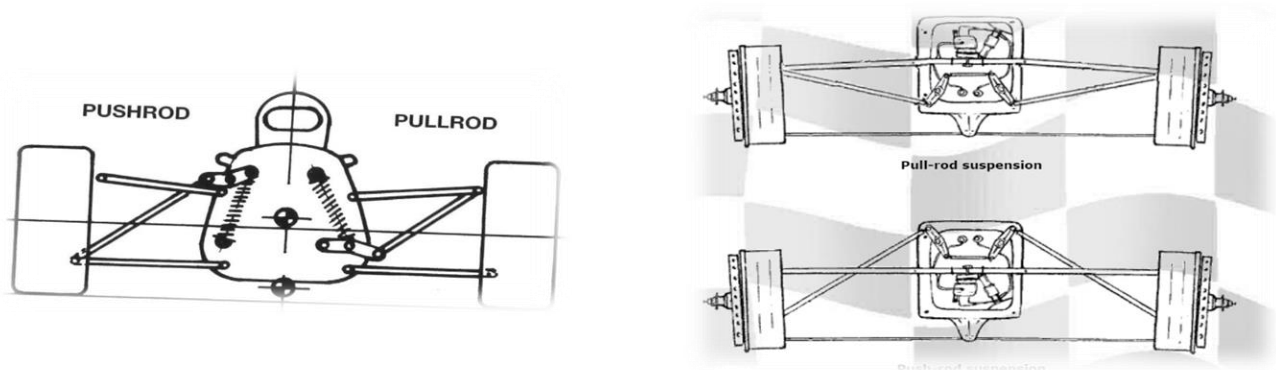


Figure 23: Push Rod and Pull Rod

Push Rod or Pull Rod				
			Options	
			Push	Pull
Criteria	Weight	Multiply	Score	Score
Low CG	80	x	8	9
Strength	100	x	5	5
Nose Height (low)	60	x	7	8
Geometry (overall)	80	x	7	5
Cost	80	x	5	5
Weight	80	x	5	5
Aerodynamics	70	x	5	6
Accessibility	60	x	8	6
			Total	
			3750	3680

Figure 24: Push or Pull Rod Design Matrix

After utilizing the multiple design iterations and design matrices for the components, I was able to design a suspension system that complies with all of the FSAE Rules and Regulations while maximizing the performance of the suspension system. A motion ratio of 2 was achieved meaning that for one inch of wheel travel, the shock travel moves one half of an inch. The movement of the shock is within the allowable shock travel so the spring will not reach its solid height.

Rear Wheel Motion & Spring Information			
Item	Symbol	Value	Units
Vertical Wheel Displacement - Bump	δ_B	1.75	in
Rear Sprung Corner Weight	$W_{s,corner}$	148.35	lb
Rear Unsprung Corner Weight	$W_{u,corner}$	11.15	lb
Bell Crank Pivot to Pushrod Length	L_1	2	in
Bell Crank Pivot to Shock Length	L_2	1	in
Bell Crank Pivot to Anti-Roll Bar Length	L_3	4	in
Uncompressed Spring Length	$L_{s,u}$	4.75	in
Maximum Shock Travel	$\Delta L_{S,MAX}$	2.25	in

Table 4: Rear Wheel Motion and Spring Info (2)

Rear Spring Length & Motion Calculations			
Item	**Symbol**	**Value**	**Units**
Initial Spring Compression	$\Delta S_{initial}$	1.58	in
Total Wheel Movement	δ_{MAX}	3.33	in
Motion Ratio	**MR**	**2.00**	**-**
Total Spring Movement	ΔS_{MAX}	**1.66**	**in**
Required Spring Rate	K_s	**376.25**	**lb/in**
Ratio - Uncompressed Spring Length to Total Spring Movement*	$R_{SL/SM}$	**2.86**	**-**

Table 5: Rear Spring Length and Motion (2)

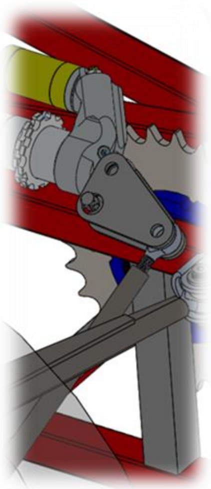


Figure 25: Final Layout (1)

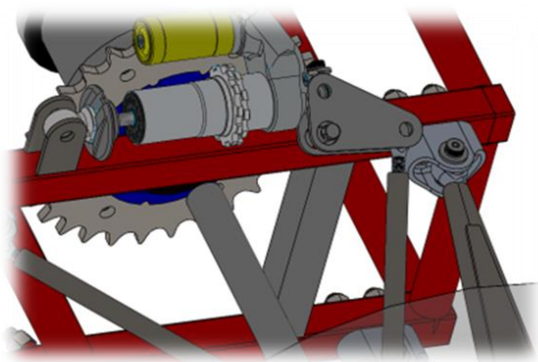


Figure 26: Final Layout (2)

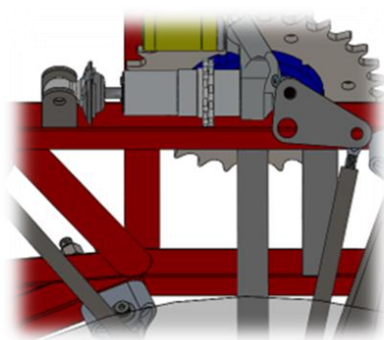


Figure 27: Final Layout (3)

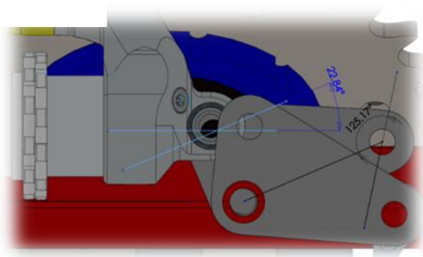


Figure 28: Maximum Jounce Angles

Figure 29: In-Plane Bell Crank Motion

B. Engineering analysis of solution

Analysis of the system solution is critical to assure that the components will withstand all the loads and stresses that it will experience during operation. Loading conditions during a race can be broken down into five categories; static, bump, acceleration, braking, and cornering. Within each condition, there are vertical, lateral, and longitudinal loads. Each case is represented by the maximum input load that it would see during operation for analysis to guarantee that the components will not fail. The load cases and their corresponding values are represented in the figure below.

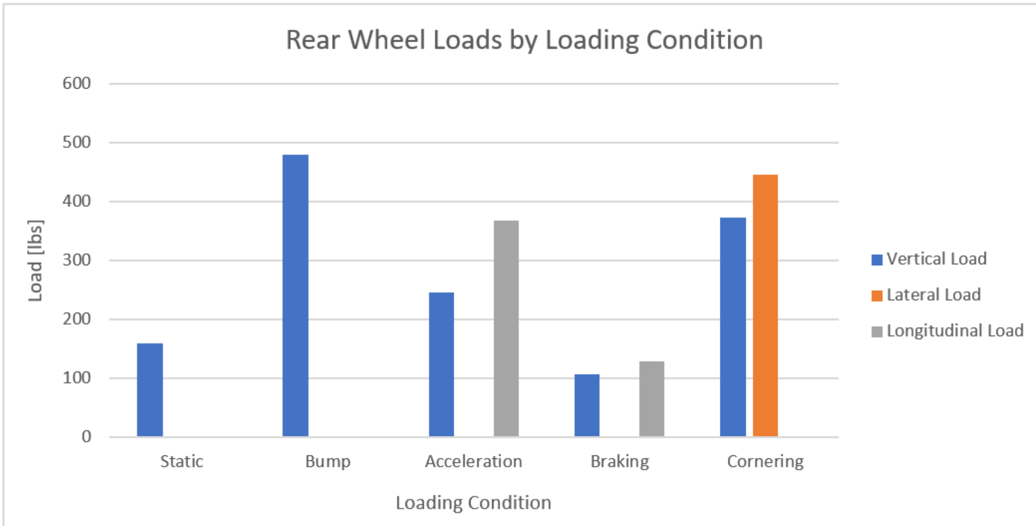


Figure 30: Loading Conditions (Rear)

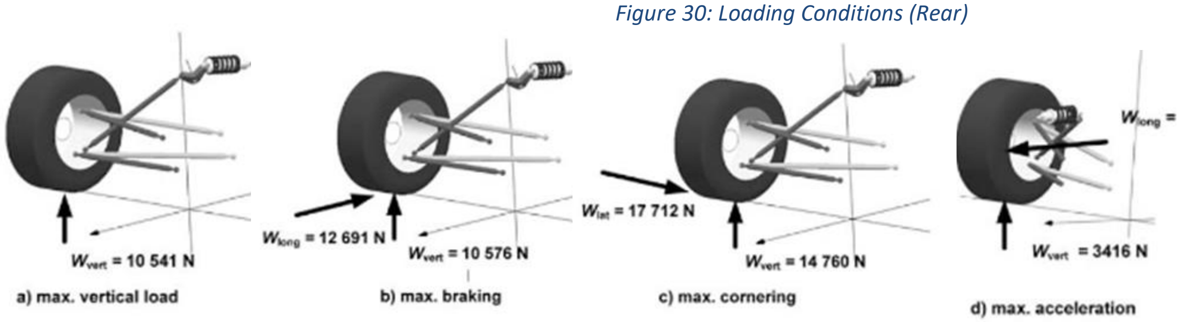


Figure 31: Max Condition Free Body Diagrams

When analyzing the rear suspension, only the bump loading and static loading are considered. All other loading is absorbed by the other members in the car. The static loading has no external forces and is considered the sprung mass of the car. The bump loading is a 3G vertical input load and is seen only on one wheel.

Load Case: Static Load		
Item	**Value**	**Units**
Rear Wheel Vertical Load	159.50	lb
Rear Wheel Lateral Load	0	lb
Rear Wheel Longitudinal Load	0	lb

Table 6: Static Load

Load Case: Maximum Vertical Load		
Item	**Value**	**Units**
Rear Wheel Vertical Load	478.50	lb
Rear Wheel Lateral Load	0	lb
Rear Wheel Longitudinal Load	0	lb

Table 7: Maximum Vertical Load

Topology Optimization, as described in the introduction, is a method of finding the best distribution of material in a design space given weight goals and constraints. This technique was used on the bell crank to reduce the weight by 50% while maintaining the stiffness in the structure. Based on the kinematics of the final design layout, loads and angles for the ride height and maximum jounce were calculated to apply as the load cases in the optimization.

Bell Crank Plate Stress Calculations			
Item	**Symbol**	**Value**	**Units**
Push Rod Input Force	F_p	396.34	lb
Shock Input Force	F_s	396.34	lb
Bell Crank Plate Thickness	t	0.125	in

Table 8: Bell Crank Stress Calculations

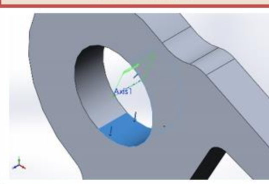
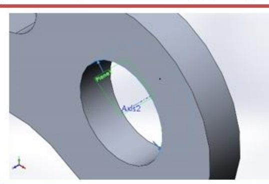
Load name	Load Image	Load Details
Force-1		Entities: 1 face(s) Type: Apply normal force Value: 396.335 lbf
Force-2		Entities: 1 face(s) Type: Apply normal force Value: 396.34 lbf

Figure 32: FEA Force Surfaces on Bell Crank

In SolidWorks, the bell crank was opened, and surfaces were split at appropriate angles for the loads to be applied normally. The material was also set to 7075 T6 Aluminum Plate. In the Simulation tab, “New Study” was chosen. In the study field, under the “Design Insight” header, is the selection of Topology Optimization. The component now needs to be fully constrained, so a bearing fixture is added at the surface of the fulcrum pin to allow the bell crank to rotate about the fulcrum axis during the optimization. Next, the loads were applied by right clicking on “Topology Study” in the design tree and “Multiple Load Case” was selected. In the load case window, the load cases for ride height and maximum jounce were applied at the shock mounting hole and the push rod mounting hole. For the optimization goal, “Best Stiffness to Weight” was selected and set to a 50% reduction of weight. This will stop the optimization once 50% weight reduction is achieved, but the added benefit is that after the optimization, a slider bar allows the user to look at any weight reduction percentage lower than 50%. Lastly, “Manufacturing Controls” were used for symmetry about the middle of the thickness and preserved regions. The preserved regions were applied at the surface of each of the mounting holes and fulcrum and adjusted to expand at a diameter of 0.05” from the surface of the holes. A mesh was also applied to complete the constraints and run the optimization.

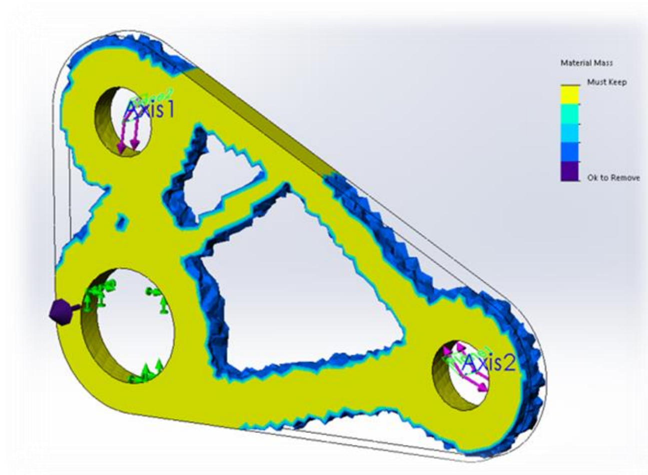


Figure 33: Max Jounce Single Load Case Optimized Bell Crank

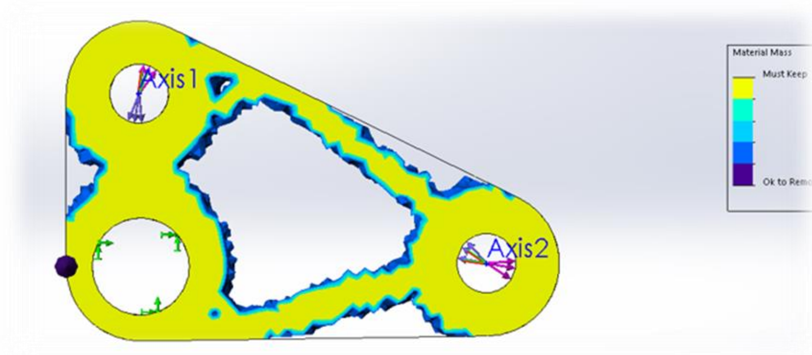


Figure 34: Multiple Load Case Optimized Bell Crank

Once the optimization is complete, the component is show as in the figures above. From the color graph on the side, all of the yellow regions must be kept in order to maintain the stiffness and the colors progressing below the yellow are of less and less significance. The part was then exported and saved as a graphic. This graphic represents the optimized part, but is not exactly manufacturable unless additive manufacturing is used. Using the graphic part, a CAD model is drawn to represent the optimized part as best as possible as seen in the figure below.

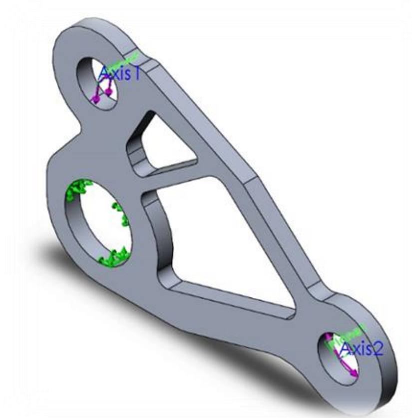


Figure 35: Optimized Bell Crank CAD Model

As seen, the chosen component was an optimized part using both the maximum load case of jounce, and the optimized component that utilized multiple load cases. This is because the support in the mid-section of the bell crank is an added benefit for the structure from an engineering stand point and does not affect the weight drastically. The final weight of the bell crank is 0.038lbs compared to the 0.16lb bell crank of the Spring 2017 suspension. This is due to the optimization and changing the material from 4130 Steel to 7075 T6 Aluminum. The weight will be significant because the suspension uses four bell cranks. To ensure that the optimized component will withstand the loading I utilized Finite Element Analysis.

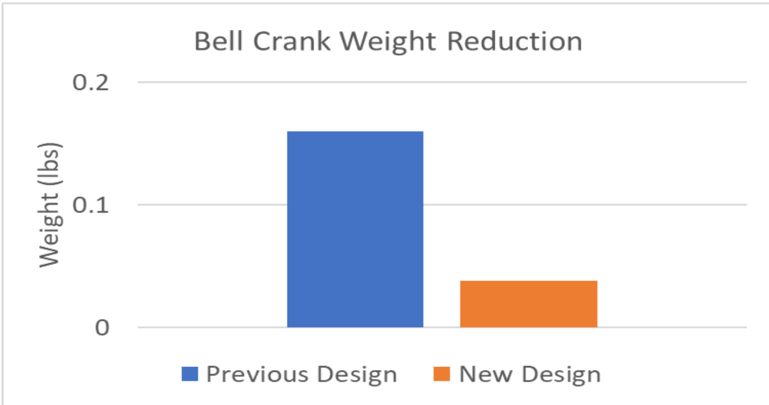


Figure 36: Bell Crank Weight Reduction

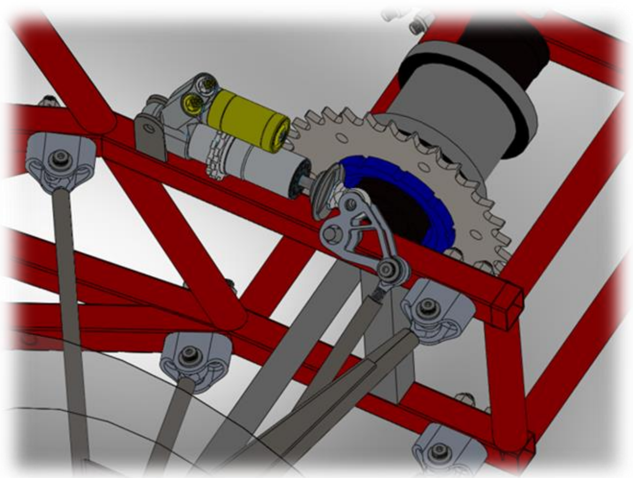


Figure 37: Optimized Bell Crank Implementation

Finite Element Analysis is a numerical method for solving engineering problems. Areas covered by FEA include structural analysis, heat transfer, mass transport, fluid flow, and electromagnetic potential. For the suspension components, structural analysis will be the area of interest.

The first FEA was run on the bell crank after the optimization. Applying the calculated forces and force angles as in the optimization study, the analysis was performed. A mesh convergence study was conducted to use the appropriate number of nodes in the mesh, so we know that our maximum stress value is accurate. In the case of the bell crank, the mesh converged at approximately 43,000 nodes. The maximum stress in the bell crank is found to be 30,840 psi while the yield stress of the 7075 T6 Aluminum is 73,000 psi. This yields a safety factor of 2.37 on this component.

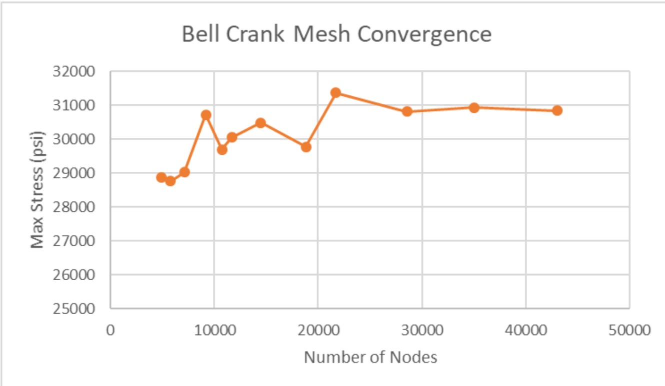


Figure 38: Bell Crank Mesh Convergence

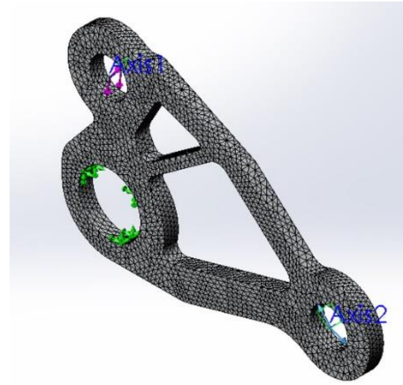


Figure 39: Bell Crank Meshed

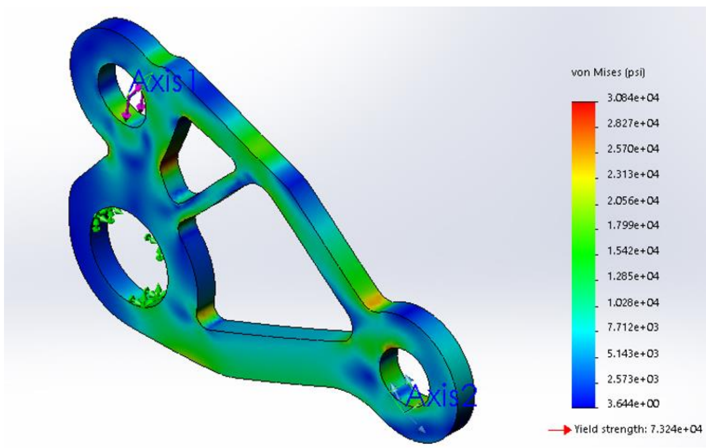
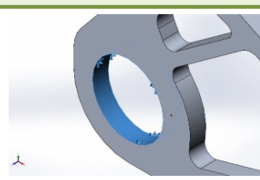


Figure 40: Bell Crank Von Mises Stresses

Using the bell crank FEA analysis, we are given the reaction forces on the fulcrum hole, which are applied to the fulcrum pin. The fulcrum pin will see two bell cranks at specific locations, so before the analysis was conducted, the bell crank location surfaces were split to apply the forces.

Fixture name	Fixture Image	Fixture Details
Fixed-1		**Entities:** 1 face(s) **Type:** Fixed Geometry

Resultant Forces

Components	X	Y	Z	Resultant
Reaction force(lbf)	-378.439	-108.488	0.0247406	393.683
Reaction Moment(lbf.in)	0	0	0	0

Figure 41: Fulcrum Hole Reaction Forces

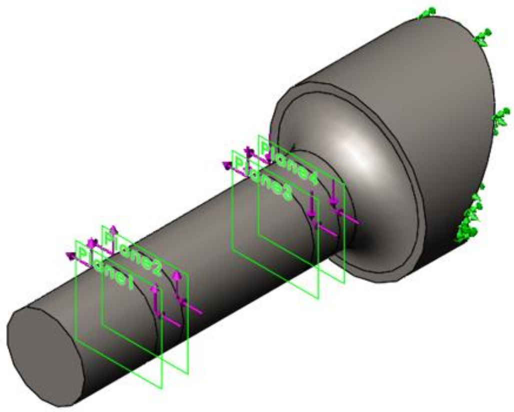


Figure 42: Fulcrum Pin Split Surfaces

Using a mesh convergence study, the mesh can be seen to converge at approximately 35,000 nodes. The fulcrum pin is made of 4340 Normalized steel and utilizing the converged mesh size gives a maximum stress of 48,000 psi. The yield stress of the steel is 103,000 psi, which means that we have a safety factor of 2.14 for the fulcrum pin.

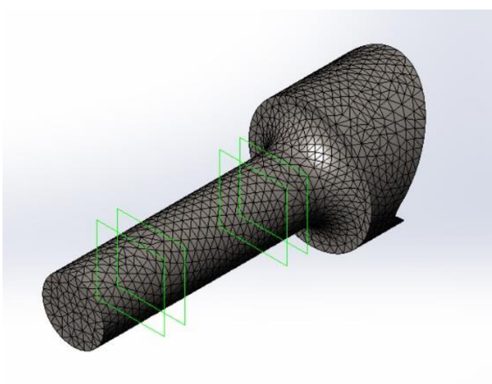


Figure 43: Fulcrum Pin Meshed

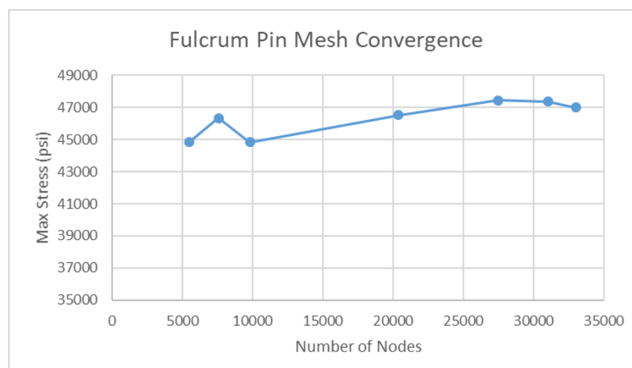


Figure 44: Fulcrum Pin Mesh Convergence

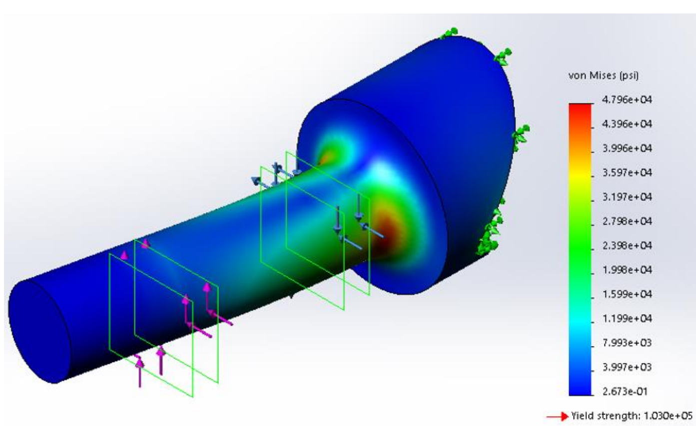


Figure 45: Fulcrum Pin Von Mises Stresses

The last FEA performed was for the shock mount. This component will see half of the shock input force since there will be two mounts absorbing the force. The mesh convergence study shows a node count of 25,000. After the analysis is complete, we have a maximum stress on the shock mounting plate to be 31,320 psi and a yield stress of 67,000 psi for 4130 Steel. This produces a factor of safety of 2.13 for the shock mount plates.

Shock Mount Plate Stress Calculations			
Item	**Symbol**	**Value**	**Units**
Shock Input Force	F_s	396.34	lb
Shock Mount Plate Thickness	t	0.125	in
Shock Shoulder Bolt Diameter	d_{bolt}	0.3125	in
Shock Shoulder Bolt Clearance Hole Wall Thickness	a	0.3285	in
Shock Mounting Hole Distance to Weld	d_1	1.25	in
Moment of Inertia	I	0.01042	in^4
Distance from Hole CL to Upper/Lower Edge	c	0.5	in
Bearing Stress from Shock	$\sigma_{bearing}$	**5073.11**	**psi**
Max Bending Stress at the Weld	$\sigma_{bending}$	**23780.20**	**psi**
Material Yield Strength - 4130 Steel	S_y	66700	psi

Table 9: Shock Mount Plate Stress Calculations

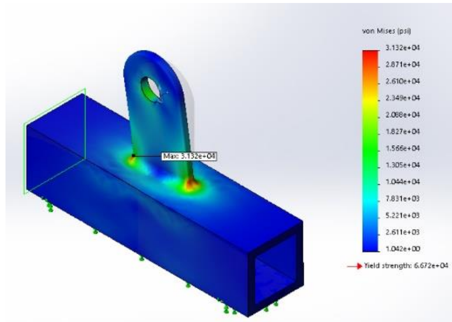


Figure 46: Shock Mount Plate Von Mises Stresses

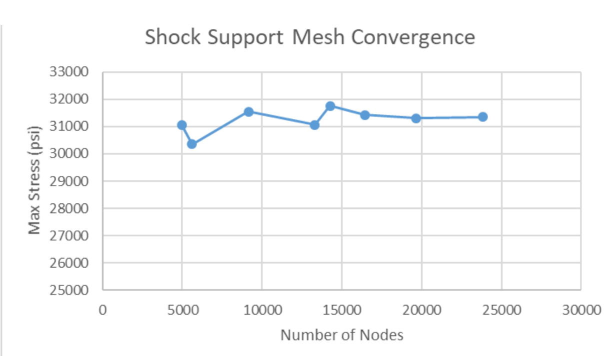


Figure 47: Shock Mount Plate Mesh Convergence

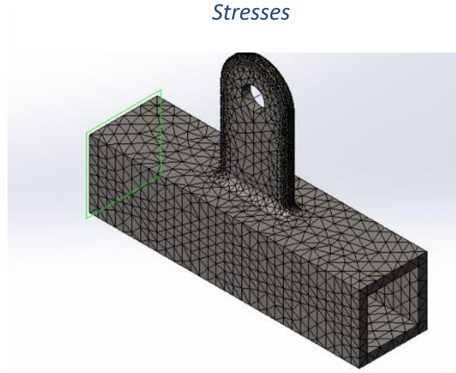


Figure 48: Shock Mount Plate Meshed

The final component to be analyzed is the push rod. FEA analysis is not exactly what needs to be considered for the push rod. This component will experience buckling, so we need to check the safety factor due to buckling using the Allowable Euler Buckling Load. The calculations are shown below. Utilizing the previous year's 4130 Steel push rod for savings on cost and manufacturing, we obtain an Allowable Euler Buckling Load of 3732lb and a Buckling Factor of Safety of 4.71. The safety factor may seem excessive, but the tube thickness and diameters are needed to fit the rod ends in properly.

Pushrod Stress Calculations			
Item	**Symbol**	**Value**	**Units**
Tube Outer Diameter	$D_{o,tube}$	0.500	in
Tube Inner Diameter	$D_{i,tube}$	0.385	in
Tube Cross-Sectional Area	$A_{c,tube}$	0.07993	in^2
Maximum Compressive Force	F_{max}	792.67	lb
Maximum Stress	σ_{max}	**9916.61**	**psi**
Material Yield Strength - 4130 Steel	S_y	66700	psi
Factor of Safety	**n**	**6.73**	-
Material Modulus of Elasticity	E	29700000	psi
Material Second Moment of Area	I	0.00198948	in^4
Pushrod Length	$L_{pushrod}$	12.5	in
Allowable Euler Buckling Load	P_b	3732.29	lb
Buckling Factor of Safety	n_b	**4.71**	-

Table 30: Pushrod Stress Calculations

C. Cost analysis of design

As stated in the project constraints, I was given a budget of $250. This money is to be utilized on the materials for the designed components, all hardware, and the manufacturing costs. In the table below is the breakdown of the money spent on componentry and labor hours and cost of manufacturing.

Cost and Budget Breakdown				
Component	**Material Cost**	**Labor Cost**	**Labor Hours**	**FSAE Material Owned**
Bell Crank	$34.36	$0	2	N
Shock Mounts	$0	$0	1	Y
Push Rod	$0	$0	0	Y
Fulcrum Pin	$19.26	$0	2	N
Total	**$53.62**	**$0**	**5**	

Table 11: Cost and Budget Breakdown

The table shows a total of $53.62 used all due to the material of the bell cranks and fulcrum pins. Material costs were not considered for the shock mounts, push rod, and hardware because the FSAE Club owned the needed material from previous years. Labor hours accrued to five total hours and a cost of zero dollars since the manufacturing was completed by me at Central Connecticut State University.

D. Project Deliverables

From the cost analysis, we see that each component was manufactured at the university. I first manufactured the bell cranks on the CNC Mill with a 3/16 endmill. I wrote the code of the toolpaths in HSMWorks in SolidWorks.

Figure 49: Bell Crank Plates In Process

Figure 50: Fabricated Bell Crank Plates

The shock mount plates were manufactured out of the 4130 steel from the previous year on the plasma router in room NC145. This method of manufacturing parts is very fast and accurate, which allows me to cut down the labor hours goal. Four shock mounts were manufactured.

Figure 51: Shock Mount Plates Fabricated

Figure 52: Shock Mount Plates on Plasma Router

The last component that was manufactured was the fulcrum pins. The pins sit on the square tube at a 53º angle. To fabricate, I first used the manual lathe to turn the 0.37in diameter pin where the bell cranks rotate. The lathe also allowed me to machine the radius between the 0.37in pin and the 0.75in base. Cutting the pin to size and facing off the base was the last step completed on the manual lathe. To make the 53º angle on the base, I used the Bridgeport mill. The fulcrum pin was positioned in the vice at the specified angle using three angle blocks; 30º, 20º, and 3º. The first pin was scrapped due to the pin moving in the vice. After making two more pins, I adjusted the way that I was cutting with the mill. Originally, I went to the depth I needed and cut sideways. The modified manufacturing method that was a success, set the X axis stationary and moved the Z axis until the required depth was reached.

Figure 53: Fulcrum Pin Held at 53 Degree Angle on Bridgeport

Figure 54: Me (Ian) Finishing Up on Bridgeport

Figure 55: Fabricated Fulcrum Pins (1)

Figure 56: Fabricated Fulcrum Pins (2)

All fabricated components were collected in a shoebox with the tools used for the manufacturing process. All hardware for the suspension system is on the current set up of the FSAE car in the FSAE room NC118.

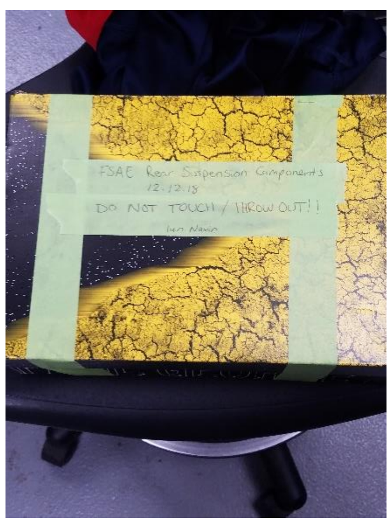


Figure 57: Shoe Box with All Suspension Components

IV. Discussion

Through the design and analysis, we can see that the suspension system is acceptable. The bell crank, fulcrum pins, shock mounts, and push rod all have a safety factor above two. This means that each component will, at the maximum, only experience a stress that is half of the yield stress. The yield stress is the elastic limit on the stress strain curve. Once a stress goes beyond the yield stress, then the component will experience plastic deformation. The components analyzed will all stay below the elastic limit at the maximum input loads and sustain life without failure.

With the new design, I was also able to maintain the in-plane bell crank motion, which only allows rotation in the bell crank and no axial loads. The motion ratio for the bell crank was increased to 2 for the vehicle giving the spring a total movement of 1.66 inches and total wheel movement of 3.33 inches when we are only required to have two inches minimum. I know from calculations from the uncompressed spring length and total spring movement ratio that I will not allow the spring to ever become solid, thus keeping optimal performance of the race car.

Relocation of the shock assembly not only eliminates all interferences with other components of the car, but its accessibility allows users to tune the suspension with ease. Another benefit is weight reduction since the old location in the rear required four steel tubes to protect and mount the shock assemblies.

Utilization of topology optimization for the bell crank was successful, though it was not the driving factor in the weight reduction. Changing material from 4130 steel to 7075 T6 aluminum was the major weight reducer. Using topology optimization for design optimization is a method that should be utilized more often, especially now that many engineering software companies are making it available. In the optimization of the bell crank, we can see that the multiple load case seems to be less stiff, but this may be because the optimization is assuming fluctuating stress between the maximum jounce and the ride height, whereas the optimization with the maximum load only is assuming the high forces constantly.

Fabrication of the components was completed on CNC Mill, plasma router, lathe, and a Bridgeport. Utilization of these machines enabled short cycle times, while precisely manufacturing the parts. Saving on time for labor allowed more thoughtful design and thorough analysis of the system. With the FSAE Club owning many of the needed materials and tools, allowed exceptional cost savings.

A. Impact Statement

The CCSU FSAE Rear Suspension Team redesigned and optimized Spring 2017 rear suspension, while packaging the suspension components within the Fall 2017 SLA Double Wishbone system. Relocation of the struts was the priority due to the interference with the new differential being implemented. A 30% weight reduction of the utilized suspension components was also desired while maintaining structural integrity. Manufacturing time and costs were desired to be reduced by 15%. Utilizing previous componentry was ideal as it leads to time and cost reductions.

Using my design and engineering knowledge, I was able to design and implement a rear suspension system building off of the SLA Double Wishbone system. Analysis of the components were completed to ensure that they will not fail during operation. The design of the system also increases the performance of the suspension system. Weight was reduced, previous componentry was utilized, money was saved, and manufacturing time was reduced.

Weight reduction in the rear suspension allows the overall weight of the vehicle to decrease, improving the performance of the race vehicle. Maintaining a low center of gravity and ride height is ideal in the performance of the vehicle as well as maintaining a lower ride frequency in the rear. The lower ride frequency allows the rear to have more mechanical grip and speed out of corners. A lighter rear allows more use of the spring travel and allows for a lower ride height which aids in the aerodynamics of the race car. Reduction in cost and labor hours allows the FSAE club to spend their money and time in more critical applications of the vehicle. A $250 budget is not very much, but a total cost of $54 is a large savings, especially when previous materials that were not utilized can be used so that there isn't scrap. Labor hours impact the manufacturer, so time savings cut costs as well as allowing the manufacturer to produce more components and profit. All design, analysis, and fabrication directly impacts the FSAE Team by saving time, money, and weight while increasing vehicle performance.

B. Future Work

Future work for the CCSU FSAE Team would include possibly implementing a sway bar (anti-roll bar) system. The importance of the sway bar is that it can reduce body roll of the vehicle in corners and irregularities in the road surface during operation. The system stabilizes the vehicle, increasing roll resistance by connecting both rear wheels with sway arms that absorb any torsion in the rear system. Implementation of the anti-roll bar will increase weight, but will increase performance. Consideration of the system will depend on if the front suspension has the system integrated. If so, it may not be completely necessary to add to the rear. Future suspension teams will need to discuss the benefits and drawbacks.

V. Conclusion/Summary

The design and results of this project directly solve the problems laid out in the problem statement of this document. The FSAE team needed a rear suspension design to be completed following the implementation of an SLA Double Wishbone Suspension system and the new differential. Desired goals included the elimination of interferences, a 30% reduction in weight, and a 15% reduction in cost and labor hours. After research and understanding of race vehicle dynamics, a system was designed. Consideration of design constraints included the 45-week time frame, $250 budget, FSAE Rules and Regulations, and previous suspension team's work.

The previous suspension system was designed very well, so optimization was performed on the system. After further research on the design of other components such as the differential, it was found that the previous suspension design was not suitable. Design matrices of the push/pull rod system and the types of bell cranks were developed using criteria that affected the concepts and performance of the design. A plated bell crank and a push rod were selected for the start of the design.

Many design set-ups were considered, with the shock assembly mounted above the differential, on the side of the vehicle sitting vertical, and on the side of the vehicle sitting horizontal. Each set-up yielded multiple forces and force angles that were found using kinematics. Optimization of the bell crank was performed on many cases, but ultimately the ideas were scrapped due to the final design system layout, which mounted the shock assembly vertically allowing exceptional accessibility for tuning. Weight was also reduced since the four protection tubes and mounting positions for the old design were eliminated. Calculations and analysis of the kinematics and dynamics of the system gave a Motion Ratio of 2, maintained the in-plane bell crank motion, and confirmed that spring travel of the strut would not lead to solid height.

Further weight reduction was achieved by changing materials of the bell crank from 4130 Steel to 7075 T6 Aluminum which is used in the aerospace industry due to its high stiffness to weight ratio. Integration of topology optimization additionally reduced weight and enabled future teams to have a tool to find the best design given a multitude of design constraints.

Knowledge of Finite Element Analysis showed maximum stresses that the components will experience during operation. Material choice and re-designs were completed when a safety factor above 1 was not met. After final design of the components and choice of material, the FEA analysis confirmed a safety factor of at least 2 for all components, concluding that the system will not fail under maximum input loads.

Fabrication of the parts utilized my knowledge of machining and also taught me about machines that I was not familiar with. The CNC Mill was new to me, but I was able to manufacture the bell cranks in a matter of two hours. A couple of problems were ran into using the HSMWorks toolpaths, but they were overcome by reprogramming. No scrap parts were produced. The speed and precision of the plasma router was ideal for cutting out the shock mounts. Set up of the process was the longest step, but once set-up, the process took only a matter of minutes. Finally, fabrication of the fulcrum pin ended in one scrap part, but lessons were learned in the method of cutting and gave two successful parts off the lathe and Bridgeport.

Costs were cut in the materials by utilizing previous componentry and materials owned by the FSAE Club. Fabricating my own parts led to zero cost in manufacturing and giving a total cost of $54. Labor hours were also reduced to 5 hours by using CNC machinery and good manufacturing planning and processes.

After completion of the design, optimization, analysis, and fabrication, all project goals were exceeded as summarized in the table below.

Goal	**Result**
Eliminate Interference	✓
Utilize Previous Components	✓
Reduce Weight by 30%	**31.5% Reduction**
Cut Costs by 15%	**78% Reduction**
Decrease Manufacturing Time by 15%	**28% Reduction**

Table 14: Goal vs. Results

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