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Mid-Semester Project
Steady State 2-D Flow in a Channel

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List of Symbols

A_N	Computational Molecule Coefficient (Northern Node)
A_P	Computational Molecule Coefficient (Primary Node)
A_S	Computational Molecule Coefficient (Southern Node)
α	Geometric Ratio of Height vs Length of Channel
B_u	Computational Molecule (See Equation 1.47)
C_1	First Integration Constant
C_2	Second Integration Constant
C	Pressure Gradient Coefficient of Computational Molecule
Δ	Delta (Difference)
δ	Boundary Layer
∂	Partial Derivative
Σ	Sum
g	Body Force
H	Height of Channel
i	Representation of the x Direction
j	Representation of the y Direction
L	Length of Channel
l	Nodal Coefficient Representing x (i) Location
m	Nodal Coefficient Representing y (j) Location
P	Pressure
p^*	Pressure (dimensionless)
ρ	Fluid Density
P_j	First Recurrence Term (TDMA)
Q_j	Second Recurrence Term (TDMA)
Re	Reynolds Number
T	Length Representation for Grid Generation
τ	Shear Stress
u	Velocity in the x Direction

u^*	Velocity in the x Direction (dimensionless)
μ	Viscosity
U	Uniform Velocity (Entrance)
v	Velocity in the y Direction
v^*	Velocity in the y Direction (dimensionless)
x	X Direction
x^*	X Direction (dimensionless)
y	Y Direction
y^*	Y Direction (dimensionless)

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Abstract

A two-dimensional, steady state, incompressible flow with a constant inlet velocity in the x direction is to be analyzed to study the behavior of the fully developed region in the form of velocity profiles. Evaluating the boundary conditions and using the finite difference and polynomial fitting methods in dimensionless forms, we found the velocity in the x direction at the center of the channel for the fully developed region 1.5. The finite difference method utilized an iterative processing on the pressure gradient to satisfy the continuity equation and the TDMA approach to step through the channel until the pressure gradient converged to approximately -12. The finite difference method produces some errors which can be seen in the approximate value of the velocity and converged pressure gradient. Verification of the finite difference method was performed using the polynomial fitting method with the second order polynomial to estimate the boundary layer. Both methods prove to be sufficient mathematic approaches to the solution.

Introduction

A steady state, two-dimensional and incompressible flow with constant properties is to be studied. This problem consists of a developing flow in a channel as depicted below:

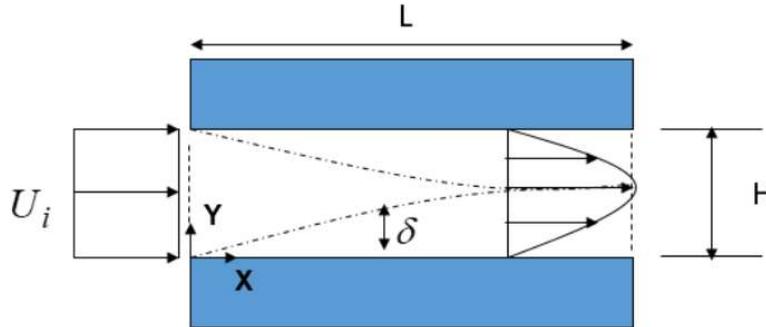


Figure 1: Developing Flow Channel

The fluid enters at a uniform velocity, U_i , and a boundary layer, δ , develops at the walls. The local velocity changing inside the boundary layer is denoted by $u(x,y)$, and the velocity in the inviscid region, close to the center line, is denoted by $U(x)$.

To analyze the flow in the developing region for $Re_H = 30$ and $\alpha = H/L = 0.1$, we will develop a computer code and compare the results with those obtained from a polynomial approximation of u/U and with commencement flow in a circular pipe.

Assumptions

To accurately solve this problem, we must consider a few assumptions. As stated in the introduction, we are assuming a steady state, 2-dimensional and incompressible flow with constant properties. The velocity at the inlet is constant, the fluid has a Reynolds number of 30 (laminar flow), and the ratio of the height of the channel is on tenth of the length.

A no slip condition is also assumed as a Dirichlet boundary condition ($u=y=0$). This means that at the solid boundary of the wall, the velocity of the fluid will be zero. Another assumption is that a maximum velocity occurs at the middle of the channel of fluid, $H/2$, so the velocity in the y direction equals zero, $\frac{du}{dy} = 0$.

For the grid generation, we will assume that the grid set-up as given will be used with $l_1 = m_1 = 11$ yielding a symmetric grid. This also allows the assumption that distance T_x and T_y are equal, as well as, Δx and Δy will be equal.

Moreover, we will derive the momentum equations in the x and y directions, but since our momentum will only occur in the x -direction, the y -direction will not be used in solving for the solution. We also will not consider the Energy Equation since we do not have any temperature or energy loss or generation in this problem.

Utilization of upwind schemes will be used for discretization which will allow replacement of one node of interest (u_p) to the next (u_e). This discretization enables determination of the velocity at discrete points (nodes) in the channel by conversion of the differential equations into algebraic

form to allow us to obtain the computational molecule for solving. This computational molecule will be defined by coefficients that can be used to solve for recurrence terms utilized in the Tri-Diagonal Matrix Algorithm (TDMA). This is a one-dimensional approach that is will be used by using the recurrence terms and back substituting to bring information from the boundary into the domain.

To begin our calculations, we will have to guess at the velocity in the x and y direction (u, v). Our guess for u will be based on the fully developed flow and the resulting pressure gradient and the v will start with -0.01 from $j > (m_1+1)/2$ and +0.01 from $j \leq (m_1-1)/2$ with $2 \leq i \leq l_1$ and $2 \leq j \leq m_2$. The reasoning for using these velocities between $j=2$ and $j=10$ is because from the given boundary conditions as stated previously, there is no velocity at the walls due to the no slip condition and a zero velocity in the center of the channel.

Conclusions

The purpose of this report is to analyze a flow in the developing region of a channel. The flow under observation is steady state, two dimensional, incompressible, with constant properties. The fluid enters the channel at a uniform velocity, U_i , and a boundary layer, δ , develops at the walls. We are given that we have a laminar flow based upon a Reynolds number of 30 and are given a ratio of the channel geometry, α , that results in 0.1. We are also given boundary conditions of a no slip condition at the wall and a zero velocity at the center of the fluid. A computer code to analyze the flow is to be developed and compared to other methods, such as polynomial fitting and commencement flow through a circular pipe, for accuracy. Governing equations of the fluid were derived and non-dimensionalized for ease of calculations.

Based on the finite difference approach in conjunction with the TDMA algorithm as seen in the analysis body of this report, the fully developed flow converges to a pressure gradient, $\frac{dp}{dx}$, of approximately -12 as seen in Figure 9. The pressure gradient lines up with the initial guess knowing that the overall global continuity must equal zero through the height of the channel as seen in Equation 1.25 – 1.30. This convergence yields a velocity of approximately 1.5 at the center of the channel, $H/2$. This is seen though each step in the mesh at the center of $y=0.5$ and in the developing flow channel graph, Figure 8.

Verification of the finite difference approach in conjunction with the TDMA algorithm is completed using the polynomial fitting approximation. Utilization of this method also yields a velocity in the x direction at the center of the fully developed profile to be 1.5. This confirms the derivations of the governing equations and finite difference equations, as well as, the computer code/algorithm generation completed in excel.

Analysis

Foundational principles must be considered and understood when approaching a fluid dynamics problem. When studying fluid dynamic problems, we consider the conservation laws which force us to recognize conservation principles. First, when we study the flow of a fluid, we consider a control volume approach that allows the fluid flow to be studied and observed within a certain region. We also recognize that we have an extensive and intensive property which are associated with and without the quantity of matter. We also must consider Reynolds transport theorem which is used to articulate the conservation laws. This theorem considers a control volume and observes what is occurring at the surface of that volume.

$$\frac{d}{dt} \int_{CM} \rho \Phi dV = \frac{\partial}{\partial t} \int_{CV} \rho \Phi dV + \frac{d}{dt} \int_S \rho \Phi (\bar{v} - \bar{v}_{ref}) dV \quad (1.0)$$

Furthermore, we can consider the Eulerian approach in computational fluid dynamics, which holds the position of the control volume stationary as the fluid particles enter and leave the control volume. The Lagrangian method, on the other hand, tracks a particle as it flows through system, rather than remaining stationary.

Solving any fluid dynamics problem yields the use of the fundamental equations of the Conservation of Mass, Momentum, and Energy. The conservation of mass is also referred to as the continuity equation which says that for constant density fluids within a control volume, the mass is conserved (what comes into the control volume must leave). The equation can be derived from the Eulerian approach

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \quad (1.1)$$

When considering a Cartesian coordinate system of (x, y, z) , we can represent the velocity vector components as (u, v, w) respectively. Equation (1.1) becomes

$$\frac{\partial \rho}{\partial x} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0 \quad (1.2)$$

We know from our assumptions that we a fluid of constant properties and that it is incompressible, making the fluid density, ρ , constant yielding

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1.3)$$

Also, from our assumptions, we are considering a two-dimensional flow in the x and y directions, which yields our final continuity equation and first Governing Equation of

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.4)$$

The Conservation of momentum is directly related to Newtons Second Law which is states that the amount of momentum within the control volume remains constant. Application of this physical law gives the momentum equation that is represented by a vector equation of the mass multiplied by the velocity. We will consider the momentum in only the x and y direction as we are in a two-dimensional domain.

$$\text{X - Momentum} \quad \frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) + \rho g_x \quad (1.5)$$

$$\text{Y - Momentum} \quad \frac{\partial v}{\partial t} + \frac{\partial v^2}{\partial y} + \frac{\partial uv}{\partial x} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_y \quad (1.6)$$

The left side of the equations represent the inertia. On the right-hand side of the equation, we have a negative pressure gradient, $\frac{\partial p}{\partial x}$, which states that we need the pressure to decrease for the fluid to accelerate, the viscosity, μ , the wall behavior from the shear stress at the walls, $\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right)$, and the body force, ρg_x .

From the momentum equations, we can derive the Governing Equation for motion as shown below. In this case, we will only consider the x-momentum as stated in the assumptions. Also, from our assumptions we have an incompressible flow with constant fluid properties, no external forces, the flow is steady state and fully developed, the velocity at the walls is zero and the velocity at $H/2$ is zero.

$$\begin{aligned} \frac{\partial}{\partial t} = 0, \quad \frac{\partial}{\partial x} = 0, \quad \frac{\partial p}{\partial y} = 0, \\ u = v = 0, \quad u = u(y) = 0 \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \end{aligned} \quad (\text{from 1.4})$$

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) + \rho g_x \quad (1.7)$$

Expanding

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) \quad (1.8)$$

Substituting Equation 1.4 into Eq. 1.8 and applying fully developed assumption

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} \right) \quad (1.9)$$

Integrating Eq. 1.9 twice yields

$$u(y) = \frac{y^2}{2\mu} \frac{dp}{dx} + yC_1 + C_2 \quad (1.10)$$

From our Dirichlet BC, no slip condition ($u=v=0$), so $C_2 = 0$.

$$\frac{du(y)}{dy} = \frac{y}{\mu} \frac{dp}{dx} + C_1 \quad (1.11)$$

We know a maximum occurs at $\pm H/2$, $\frac{\partial u}{\partial y} = 0$ and at $y = H/2$, so $C_1 = -\frac{H}{2\mu} \frac{dp}{dx}$

$$u(y) = \frac{1}{2\mu} \frac{dp}{dx} (y^2 - Hy) \quad (1.12)$$

Non-dimensionalization is a common method used to simplify equations that may have defining physical dimensions. Non-dimensionalization can be completed by scaling the values relative to a specific unit. Non-dimensionalization of current variables can be seen below. Note that all dimensionless variables are denoted by a superscript, “*”.

$$y^* = \frac{y}{H} \rightarrow y = y^*H \quad (1.13)$$

$$x^* = \frac{x}{L} \rightarrow x = x^*L \quad (1.14)$$

$$u^* = \frac{u}{U} \rightarrow u = u^*U \quad (1.15)$$

$$v^* = \frac{v}{U\alpha} \rightarrow v = v^*U\alpha \quad (1.16)$$

$$p^* = \frac{(p)H^2}{\mu UL} \rightarrow p = \frac{p^*\mu UL}{H^2} \quad (1.17)$$

A note on scaling of the pressure term is that there are two different approaches due to the references though they use the same force equation. One is used for creeping flows and the other for fast flows. In our case, we have a low Reynolds number of 30, which is considered a creeping flow. With the creeping flow, we consider the pressure in relation to the viscous effects and use a normalization factor of $\frac{\mu U}{L}$ rather than the pressure in relation to inertia and use of a normalization factor of ρU^2 in a fast flow. One must be aware of the Reynolds number of the flow of interest when obtaining the dimensionless equation for pressure.

With the non-dimensionalization of our variables, we can visualize our boundary conditions as seen in Figure 2.

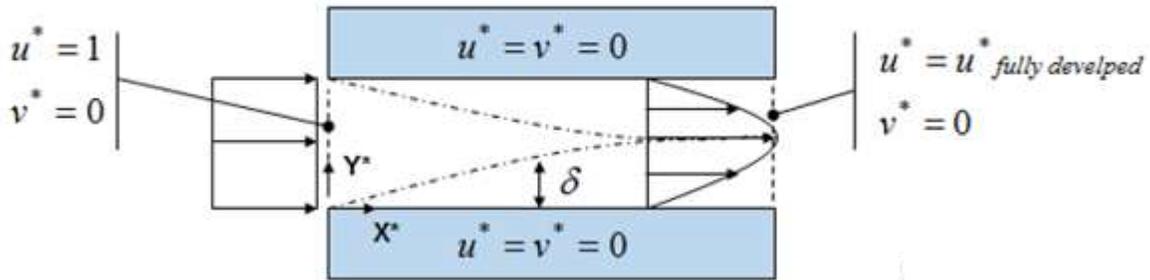


Figure 2: Non-Dimensionalized Boundary Conditions

Now that we have non-dimensionalized variables, we can non-dimensionalize equations as well. Non-dimensionalized form of Eq. 1.12 becomes

$$u^*U = \frac{(y^*H)^2}{2\mu} * \frac{\partial p^*\mu UL}{H^2 \partial x^*L} - \frac{(y^*H)^2}{2\mu} * \frac{\partial p^*\mu UL}{H^2 \partial x^*L} \quad (1.18)$$

$$u^* = \frac{(y^*)^2}{2} * \frac{\partial p^*}{\partial x^*} - \frac{(y^*)^2}{2} * \frac{\partial p^*}{\partial x^*} \quad (1.19)$$

$$u^* = \frac{1}{2} \frac{\partial p^*}{\partial x^*} (y^{*2} - y^*) \quad (1.20)$$

Knowing our Governing Equation of motion, the mesh/grid set up must be accomplished for moving forward in the problem. The mesh set-up, as recommended is shown below:

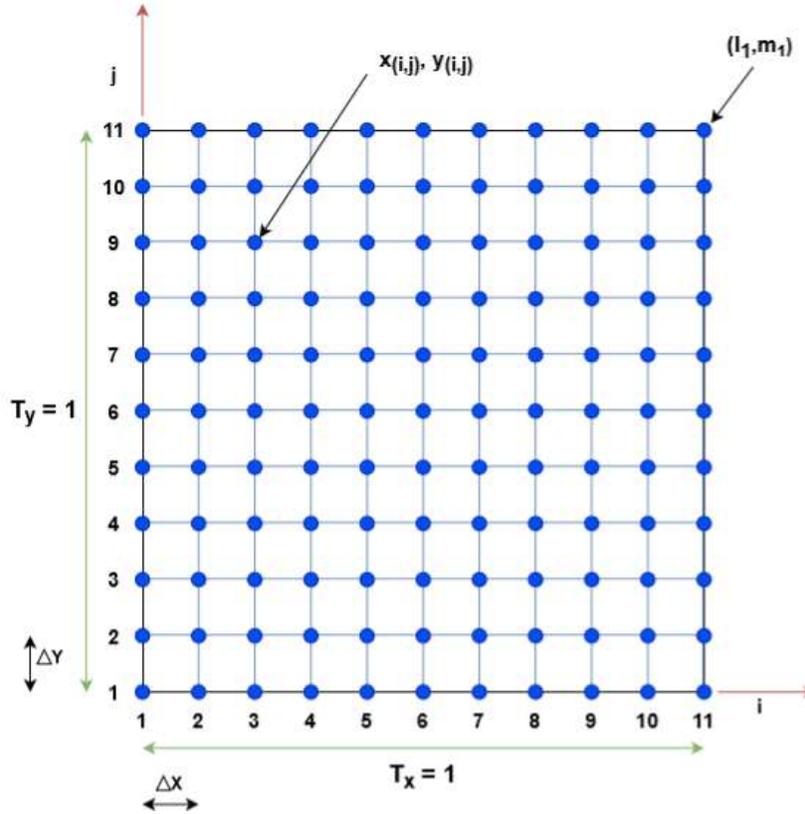


Figure 3: Grid Generation

For the grid we let $l_1 = m_1 = 11$. This allows us to obtain our Δx and Δy values to add to our know values. The equations are as follows

$$\Delta x = \frac{T_x}{l_1 - 1} \quad (1.21)$$

$$\Delta x = \frac{T_x}{11 - 1} = 0.1 \quad (1.22)$$

$$\Delta y = \frac{T_y}{m_1 - 1} \quad (1.23)$$

$$\Delta y = \frac{T_y}{11 - 1} = 0.1 \quad (1.24)$$

Obtaining these values allows for a complete set-up table.

Re_H	30
α	0.1
T_x	1
T_y	1
Δx	0.1
Δy	0.1

Given in problem statement

Table 1: Set-Up Table

To begin to find a solution to the channel flow, we start with an initial guess for u^* observing a global continuity through the pressure gradient. We know that due to overall continuity, the sum of all $u^* dy^*$ in the y direction of the channel (from wall to wall) is equal to one.

$$\int_0^1 u^* dy^* = 1 \quad (1.25)$$

Substituting Eq. 1.20 into Eq. 1.25

$$1 = \int_0^1 \frac{1}{2} \frac{dp^*}{dx^*} y^{*2} dy^* - \int_0^1 \frac{1}{2} \frac{dp^*}{dx^*} y^* dy^* \quad (1.26)$$

$$1 = \frac{1}{2} \frac{dp^*}{dx^*} \int_0^1 y^{*2} dy^* - \frac{1}{2} \frac{dp^*}{dx^*} \int_0^1 y^* dy^* \quad (1.27)$$

$$1 = \frac{1}{2} \frac{dp^*}{dx^*} \left[\frac{y^3}{3} \right]_0^1 - \frac{1}{2} \frac{dp^*}{dx^*} \left[\frac{y^2}{2} \right]_0^1 \quad (1.28)$$

$$1 = \frac{1}{2} \frac{dp^*}{dx^*} \left(\frac{-1}{6} \right) \quad (1.29)$$

$$\frac{dp^*}{dx^*} = -12 \quad (1.30)$$

For the fully developed flow, we now know that the pressure gradient will be -12 and this pressure gradient will give us our velocity values when the flow is fully developed.

To obtain the governing equations in a dimensionless form that is useful, we must take the x-momentum equation from Eq. 1.8 and substitute in given and dimensionless values. We are given the Reynolds number of the fluid, so that will be incorporated.

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) \quad (\text{from 1.8})$$

$$\rho \left[u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} \right) \quad (1.31)$$

$$\rho \left[u^* U \frac{\partial(u^* U)}{\partial(x^* L)} + v^* U \alpha \frac{\partial(u^* U)}{\partial(y^* H)} \right] = -\frac{\partial p^*}{\partial x^*} * \frac{\mu U}{H^2} + \mu \left(\frac{\partial^2(u^* U)}{\partial(y^* H)^2} + \frac{\partial^2(u^* U)}{\partial(x^* L)^2} \right) \quad (1.32)$$

$$\frac{\rho U H^2}{\mu} \left[u^* \frac{\partial(u^*)}{\partial(x^* L)} + v^* * \frac{H}{L} \frac{\partial(u^*)}{\partial(y^* H)} \right] = -\frac{\partial p^*}{\partial x^*} + \mu \left(\frac{\partial^2(u^*)}{\partial(y^*)^2} + \frac{\partial^2(u^*) H^2}{\partial(x^* L)^2} \right) \quad (1.33)$$

$$\frac{\rho U H^2}{\mu} \left[u^* \frac{\partial(u^*)}{\partial(x^*)} + v^* \frac{\partial(u^*)}{\partial(y^*)} \right] = -\frac{\partial p^*}{\partial x^*} + \left(\frac{\partial^2(u^*)}{\partial(y^*)^2} + \frac{\partial^2(u^*) H^2}{\partial(x^* L)^2} \right) \quad (1.34)$$

Substituting in the Reynolds number and knowing $\alpha = H/L$ yields

$$Re_H \alpha \left[u^* \frac{\partial(u^*)}{\partial(x^*)} + v^* \frac{\partial(u^*)}{\partial(y^*)} \right] = -\frac{\partial p^*}{\partial x^*} + \frac{\partial^2(u^*)}{\partial(y^*)^2} + \alpha^2 \left(\frac{\partial^2(u^*)}{\partial(x^*)^2} \right) \quad (1.35)$$

Similarly, for y-momentum.

All values in the x-momentum equation (Eq. 1.35) are known except for $\frac{\partial(u^*)}{\partial(y^*)}$ and $\frac{\partial^2(u^*)}{\partial(y^*)^2}$. These unknowns can be represented by first and second order finite difference schemes of a uniform grid. The finite difference scheme transforms the differential equations into algebraic form using the Taylor series and assessing the node of interest and surrounding nodes. After assessment, the node of interest will move through the grid using the upwind scheme. The node of interest and surrounding nodes can be viewed as shown below

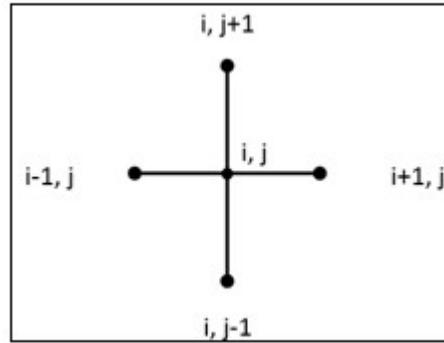


Figure 4: Node of Interest View

Viewing the node of interest as in Figure 4, the discretization equations become

$$\frac{\partial(u^*)}{\partial(y^*)} = (u_{i,j+1}^* - u_{i,j-1}^*) / \Delta y^* \quad (1.36)$$

$$\frac{\partial^2(u^*)}{\partial(y^*)^2} = (u_{i,j+1}^* - u_{i,j-1}^* - 2u_{i,j}^*) / \Delta y^{*2} \quad (1.37)$$

$$\frac{\partial(u^*)}{\partial(x^*)} = (u_{i+1,j}^* - u_{i-1,j}^*) / \Delta x^* \quad (1.38)$$

$$\frac{\partial^2(u^*)}{\partial(x^*)^2} = (u_{i+1,j}^* - u_{i-1,j}^* - 2u_{i,j}^*) / \Delta x^{*2} \quad (1.39)$$

If we rewrite the x-momentum equation as

$$\frac{\partial^2(u^*)}{\partial(y^*)^2} - (Re_H \alpha v^*) \frac{\partial(u^*)}{\partial(y^*)} = [(Re_H \alpha u^*) \frac{\partial(u^*)}{\partial(x^*)} - \alpha^2 \left(\frac{\partial^2(u^*)}{\partial(x^*)^2} \right) + \frac{\partial p^*}{\partial x^*}] \quad (1.40)$$

We now discretize the x-momentum equation using the finite difference equations of the unknowns and the upwind scheme for the right hand side of the equation and replace the pressure gradient, $\frac{\partial p^*}{\partial x^*}$, with a variable, C .

$$\frac{(u_{i,j+1}^* - u_{i,j-1}^* - 2u_{i,j}^*)}{\Delta y^{*2}} - (Re_H \alpha v^*) \frac{(u_{i,j+1}^* - u_{i,j-1}^*)}{\Delta y^*} = (Re_H \alpha u^*) \frac{(u_{i+1,j}^* - u_{i-1,j}^*)}{\Delta x^*} - \alpha^2 \frac{(u_{i+1,j}^* - u_{i-1,j}^* - 2u_{i,j}^*)}{\Delta x^{*2}} + C \quad (1.41)$$

Simplifying

$$\frac{(-2)}{\Delta y^{*2}} u_{i,j}^* = \left(\frac{(Re_H \alpha v^*)}{\Delta y^*} - \frac{1}{\Delta y^{*2}} \right) u_{i,j+1}^* + \left(-\frac{(Re_H \alpha v^*)}{\Delta y^*} - \frac{1}{\Delta y^{*2}} \right) u_{i,j-1}^* + \left[(Re_H \alpha u^*) \frac{(u_{i+1,j}^* - u_{i-1,j}^*)}{\Delta x^*} - \alpha^2 \frac{(u_{i+1,j}^* - u_{i-1,j}^* - 2u_{i,j}^*)}{\Delta x^{*2}} \right] + C \quad (1.42)$$

When discretizing and viewing the node of interest and surrounding nodes, we can also represent the positions as north, east, south, west, and primary

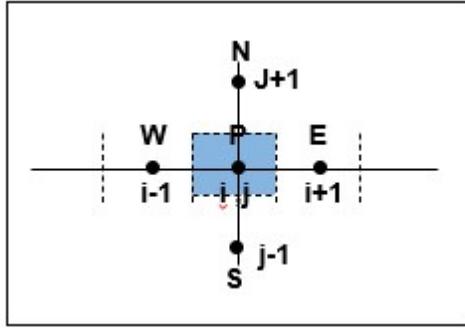


Figure 5: N, E, S, W, P Nodal Representation

Equation 1.42 can be simplified using variable representation in conjunction with the new nodal representation which becomes our computational molecule.

$$A_p u_p = A_S u_S + A_N u_N + B_u + C \quad (1.43)$$

Where

$$A_N = \frac{(Re_H \alpha v^*)}{\Delta y^*} - \frac{1}{\Delta y^{*2}} \quad (1.44)$$

$$A_S = -\frac{(Re_H \alpha v^*)}{\Delta y^*} - \frac{1}{\Delta y^{*2}} \quad (1.45)$$

$$A_p = A_N A_S \quad (1.46)$$

$$B_u = \left[(Re_H \alpha u_p) \frac{(u_P - u_W)}{\Delta x^*} - \alpha^2 \frac{(u_E - u_W - 2u_P)}{\Delta x^{*2}} \right] \quad (1.47)$$

$$D = B_u + C \quad (1.48)$$

$$C = \frac{\Delta p^*}{\Delta x^*} \quad (1.49)$$

Using the one-dimensional TDMA solution method, we can selectively view one step in the geometry grid that will look like

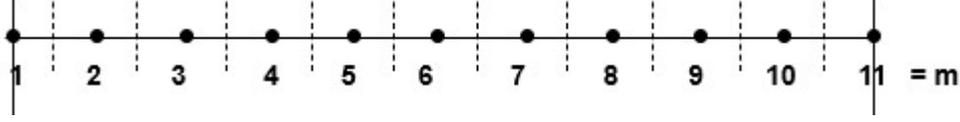


Figure 6: TDMA 1D View

The dependent variable of Eq. 1.43 is u where we can find

$$u_{i,j} = P_j u_{i,j+1} + Q_j \quad (1.50)$$

using our boundary point equations of $A_N = 0$ and $A_m = 0$. Knowing $u_{i,j-1} = P_{i,j-1} u_{i,j} + Q_{j-1}$, we can plug in $u_{i,j-1}$ into Eq 1.43 and obtain

$$A_p u_{i,j} = A_N u_{i,j+1} + A_S (P_{i,j-1} u_{i,j} + Q_{j-1}) + D \quad (1.51)$$

Solving for P_j and Q_j , which are our recurrence terms, we obtain

$$P_j = \frac{A_N}{A_P - A_S P_{j-1}} \quad (1.52)$$

$$Q_j = \frac{D + A_S Q_{j-1}}{A_P - A_S P_{j-1}} \quad (1.53)$$

Using these recurrence terms as we move through the grid will allow us to solve for the velocity in the x and y direction locally and then globally as the sum of the local velocities. Using the continuity equation $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$, we can solve for the local velocity in the y direction by

$$v^* = - \int_0^{\Delta y} \frac{\partial u^*}{\partial x^*} dy^* \quad (1.54)$$

$$v_{i,j} = - \left(\frac{u_{i,j} - u_{i-1,j}}{\Delta x} \right) \Delta y \quad (1.55)$$

The $u_{i,j}$ and $v_{i,j}$ of the specific step will be used as the new upstream conditions for the next step, ($u_P = u_E$) due to the upwind scheme. The overall or global continuity can be solved by

$$\int_0^1 u^* dy^* = \sum_{j=1}^{m-1} u_{i,j} \Delta y = 1 \quad (1.56)$$

At each step the pressure gradient will be found by finding the value of C that makes the mass = 1 along the height of the channel at that particular step. The steps will continue to iterate on C , until the mass flow converges.

Using our uniform mesh/grid, inlet conditions, boundary conditions, computational molecule, and recurrence coefficients, we set up our code in a spread sheet. We can first start off with our initial

guess of the velocity in the y direction of 0.01 from a step in from each wall. At the walls and center of the channel, we have a y velocity of 0. From our initial guess, we have a pressure gradient of -12. This initial guess is at position $x^*=0$ and we monitor the values along the y^* direction.

delta x	delta y	Re	Alpha		dp/dx Iteration for Mass Conservation,												
0.1	0.1	30	0.1		-22.82312463												

			Inlet		Closed Form with dp/dx = -12		Guess		Coefficients					P and Q Terms		TDMA Solution i=1	
x^*	y^*	$u^*\text{deltay}$	$u^*(x=0)$	$v^*(x=0)$	$u^*(x=0.1)$	$v^*(x=0.1)$	A_n	A_s	A_p	B_u	D	P	Q	u^*	v^*		
0.1	1	0.1	1	0	0	0	0	0	1	0	0	0	0	0	0		
	0.9	0.1	1	0	0.54	0.01	-99.7	-100.3	-200	-7.912	-30.735	0.4985	0.15368	0.702	0.1		
	0.8	0.1	1	0	0.96	0.01	-99.7	-100.3	-200	-1.192	-24.015	0.66466	0.26286	1.100	0.1		
	0.7	0.1	1	0	1.26	0.01	-99.7	-100.3	-200	10.088	-12.735	0.74775	0.29325	1.260	0.1		
	0.6	0.1	1	0	1.44	0.01	-99.7	-100.3	-200	19.448	-3.3751	0.79759	0.2623	1.292	0.1		
	0.5	0.1	1	0	1.5	0	-100	-100	-200	23	0.17688	0.83167	0.21667	1.292	0		
	0.4	0.1	1	0	1.44	-0.01	-100.3	-99.7	-200	19.448	-3.3751	0.85666	0.21333	1.292	-0.1		
	0.3	0.1	1	0	1.26	-0.01	-100.3	-99.7	-200	10.088	-12.735	0.87528	0.29674	1.260	-0.1		
	0.2	0.1	1	0	0.96	-0.01	-100.3	-99.7	-200	-1.192	-24.015	0.8897	0.47546	1.100	-0.1		
	0.1	0.1	1	0	0.54	-0.01	-100.3	-99.7	-200	-7.912	-30.735	0.9012	0.70207	0.702	-0.1		
	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0		
Mass=		1											Mass=	1			

Figure 7: Sample Coding $x^*=0$ and $x^*=0.1$

As seen, with our initial guess of a pressure gradient of -12, we have a symmetric flow field starting at zero on the walls (no slip boundary condition) and we increase towards the middle to 1.5. When looking for the correct velocities at $x^*=0.1$, we iterate the dp/dx for mass convergence of the u^* values to be equal to 1. This occurs at -22.823.

We now use the u^* and v^* values for this step and apply them to the next step of $x^*=0.2$. Plugging in all of our equations to find the coefficients of the computational molecule yields our P and Q values, that allow us to back substitute and obtain values of the velocities in the TDMA. The B_u value is very important as using the wrong nodal values will result in incorrect answers. For example, for $x^*=0.2$, our u_p is the u^* from $x=0.1$ and the u_w is the initial guess u^* of $x=0$. When moving on to $x^*=0.3$, the u_p is the u^* from $x=0.2$ and the u_w is the u^* from $x=0.1$. This pattern continues through all the steps. The v^* value is also important to calculate at it is used on the future steps. For example, to calculate the v^* for $x^*=0.3$, we use the u^* from $x^*=0.3$ and the u^* from $x^*=0.2$. One importance is that the v^* will be symmetric, so anything positive on one side of the channel will be negative on the opposite side of the channel and vice versa. Each step is iterated on the pressure gradient until its' u^* mass converges to 1. The remainder of the cells for each step are shown below.

delta x	delta y	Re	Alpha
0.1	0.1	30	0.1

dp/dx Iteration for Mass Conservation, C
-22.82312463

			Inlet		Closed Form with dp/dx = -12		Guess		Coefficients					P and Q Terms		TDMA Solution i=1	
x*	y*	u*deltay	u*(x=0)	v*(x=0)	u*(x=0.1)	v*(x=0.1)	An	As	Ap	Bu	D	P	Q	u*	v*		
0.1	1	0.1	1	0	0	0	0	0	1	0	0	0	0	0	0		
	0.9	0.1	1	0	0.54	0.01	-99.7	-100.3	-200	-7.912	-30.735	0.499	0.154	0.702	0.1		
	0.8	0.1	1	0	0.96	0.01	-99.7	-100.3	-200	-1.192	-24.015	0.665	0.263	1.100	0.1		
	0.7	0.1	1	0	1.26	0.01	-99.7	-100.3	-200	10.088	-12.735	0.748	0.293	1.260	0.1		
	0.6	0.1	1	0	1.44	0.01	-99.7	-100.3	-200	19.448	-3.375	0.798	0.262	1.292	0.1		
	0.5	0.1	1	0	1.5	0	-100	-100	-200	23.000	0.177	0.832	0.217	1.292	0		
	0.4	0.1	1	0	1.44	-0.01	-100.3	-99.7	-200	19.448	-3.375	0.857	0.213	1.292	-0.1		
	0.3	0.1	1	0	1.26	-0.01	-100.3	-99.7	-200	10.088	-12.735	0.875	0.297	1.260	-0.1		
	0.2	0.1	1	0	0.96	-0.01	-100.3	-99.7	-200	-1.192	-24.015	0.890	0.475	1.100	-0.1		
	0.1	0.1	1	0	0.54	-0.01	-100.3	-99.7	-200	-7.912	-30.735	0.901	0.702	0.702	-0.1		
	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0		
	Mass=	1											Mass=	1			

Table 2: Initial Step Code (x*=0, x*=0.1)

dp/dx
Iteration for
Mass
Conservation,
C
 -20.89628873

			Inlet		TDMA Solution i=1		Coefficients					P and Q Terms		TDMA Solution i=2	
x*	y*	u*deltay	u*(x=0)	v*(x=0)	u*	v*	An	As	Ap	Bu	D	P	Q	u*	v*
0.2	1	0.1	1	0	0.000	0	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.702	0.1	-97	-103	-200	-6.5729	-27.4692	0.485	0.137346	0.632	0.070
	0.8	0.1	1	0	1.100	0.1	-97	-103	-200	3.403665	-17.4926	0.646473	0.210865	1.019	0.081
	0.7	0.1	1	0	1.260	0.1	-97	-103	-200	10.07144	-10.8248	0.727064	0.243933	1.250	0.009
	0.6	0.1	1	0	1.292	0.1	-97	-103	-200	11.6301	-9.26619	0.775303	0.274883	1.384	-0.092
	0.5	0.1	1	0	1.292	0	-100	-100	-200	11.58719	-9.3091	0.816528	0.300462	1.430	0.000
	0.4	0.1	1	0	1.292	-0.1	-103	-97	-200	11.6301	-9.26619	0.852672	0.31798	1.384	0.092
	0.3	0.1	1	0	1.260	-0.1	-103	-97	-200	10.07144	-10.8248	0.878159	0.355262	1.250	-0.009
	0.2	0.1	1	0	1.100	-0.1	-103	-97	-200	3.403665	-17.4926	0.897067	0.452479	1.019	-0.081
	0.1	0.1	1	0	0.702	-0.1	-103	-97	-200	-6.5729	-27.4692	0.91163	0.631588	0.632	-0.070
	0	0.1	1	0	0.000	0	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 3: Initial Step Code (x*=0.2)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p>
-13.88865853

			Inlet		TDMA Solution i=2		Coefficients					P and Q Terms		TDMA Solution i=3	
x^*	y^*	$u^*\Delta y$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	A_n	A_s	A_p	B_u	D	P	Q	u^*	v^*
0.3	1	0.1	1	0	0.000	0.000	0.0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.632	0.070	-97.9	-102.115	-200	-1.406	-15.295	0.489	0.076	0.564	0.067
	0.8	0.1	1	0	1.019	0.081	-97.6	-102.431	-200	-2.559	-16.447	0.651	0.162	0.996	0.023
	0.7	0.1	1	0	1.250	0.009	-99.7	-100.285	-200	-0.365	-14.254	0.740	0.226	1.282	-0.031
	0.6	0.1	1	0	1.384	-0.092	-102.7	-97.2539	-200	3.892	-9.997	0.803	0.250	1.426	-0.042
	0.5	0.1	1	0	1.430	0.000	-100.0	-100	-200	6.103	-7.786	0.835	0.274	1.464	0.000
	0.4	0.1	1	0	1.384	0.092	-97.3	-102.746	-200	3.892	-9.997	0.852	0.334	1.426	0.042
	0.3	0.1	1	0	1.250	-0.009	-100.3	-99.7153	-200	-0.365	-14.254	0.871	0.413	1.282	0.031
	0.2	0.1	1	0	1.019	-0.081	-102.4	-97.5687	-200	-2.559	-16.447	0.891	0.494	0.996	-0.023
	0.1	0.1	1	0	0.632	-0.070	-102.1	-97.8854	-200	-1.406	-15.295	0.905	0.564	0.564	-0.067
	0	0.1	1	0	0.000	0.000	0.0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 4: Initial Step Code ($x^*=0.3$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p>
-13.04608811

			Inlet		TDMA Solution i=3		Coefficients					P and Q Terms		TDMA Solution i=4	
x^*	y^*	$u^*\text{deltay}$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	A_n	A_s	A_p	B_u	D	P	Q	u^*	v^*
0.4	1	0.1	1	0	0.000	0.000	0.00	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.564	0.067	-97.98	-102.023	-200	-1.21	-14.26	0.49	0.07	0.550	0.014
	0.8	0.1	1	0	0.996	0.023	-99.32	-100.679	-200	-0.70	-13.75	0.66	0.14	0.978	0.018
	0.7	0.1	1	0	1.282	-0.031	-100.94	-99.0553	-200	1.24	-11.80	0.75	0.19	1.273	0.008
	0.6	0.1	1	0	1.426	-0.042	-101.25	-98.7517	-200	1.82	-11.22	0.80	0.24	1.446	-0.021
	0.5	0.1	1	0	1.464	0.000	-100.00	-100	-200	1.53	-11.52	0.84	0.30	1.504	0.000
	0.4	0.1	1	0	1.426	0.042	-98.75	-101.248	-200	1.82	-11.22	0.86	0.36	1.446	0.021
	0.3	0.1	1	0	1.282	0.031	-99.06	-100.945	-200	1.24	-11.80	0.87	0.42	1.273	-0.008
	0.2	0.1	1	0	0.996	-0.023	-100.68	-99.3205	-200	-0.70	-13.75	0.89	0.49	0.978	-0.018
	0.1	0.1	1	0	0.564	-0.067	-102.02	-97.9766	-200	-1.21	-14.26	0.90	0.55	0.550	-0.014
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 5: Initial Step Code ($x^*=0.4$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p>
-12.55430888

			Inlet		TDMA Solution i=4		Coefficients					P and Q Terms		TDMA Solution i=5	
x^*	y^*	$u^*\Delta y$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	A_n	A_s	A_p	B_u	D	P	Q	u^*	v^*
0.5	1	0.1	1	0	0.000	0.000	0.000	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.550	0.014	-99.590	-100.41	-200	-0.24	-12.79	0.50	0.06	0.550	0.000
	0.8	0.1	1	0	0.978	0.018	-99.454	-100.546	-200	-0.55	-13.11	0.66	0.13	0.976	0.002
	0.7	0.1	1	0	1.273	0.008	-99.752	-100.248	-200	-0.32	-12.88	0.75	0.19	1.276	-0.002
	0.6	0.1	1	0	1.446	-0.021	-100.616	-99.384	-200	0.91	-11.64	0.80	0.25	1.447	-0.001
	0.5	0.1	1	0	1.504	0.000	-100.000	-100	-200	1.81	-10.75	0.83	0.29	1.501	0.000
	0.4	0.1	1	0	1.446	0.021	-99.384	-100.616	-200	0.91	-11.64	0.86	0.36	1.447	0.001
	0.3	0.1	1	0	1.273	-0.008	-100.248	-99.752	-200	-0.32	-12.88	0.87	0.42	1.276	0.002
	0.2	0.1	1	0	0.978	-0.018	-100.546	-99.4538	-200	-0.55	-13.11	0.89	0.49	0.976	-0.002
	0.1	0.1	1	0	0.550	-0.014	-100.410	-99.5901	-200	-0.24	-12.79	0.90	0.55	0.550	0.000
0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000	
	Mass=	1											Mass=	1	

Table 6: Initial Step Code ($x^*=0.5$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p> <p style="text-align: center;">-12.1256972</p>

			Inlet		TDMA Solution i=5		Coefficients					P and Q Terms		TDMA Solution i=6	
x^*	y^*	$u^*\text{deltay}$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	An	As	Ap	Bu	D	P	Q	u^*	v^*
0.6	1	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.550	0.000	-99.992	-100.008	-200	-0.005	-12.130	0.500	0.061	0.545	0.005
	0.8	0.1	1	0	0.976	0.002	-99.948	-100.052	-200	-0.052	-12.178	0.666	0.122	0.970	0.007
	0.7	0.1	1	0	1.276	-0.002	-100.068	-99.9319	-200	0.089	-12.037	0.750	0.181	1.272	0.003
	0.6	0.1	1	0	1.447	-0.001	-100.033	-99.9669	-200	0.049	-12.077	0.800	0.242	1.455	-0.007
	0.5	0.1	1	0	1.501	0.000	-100.000	-100	-200	-0.127	-12.253	0.833	0.304	1.516	0.000
	0.4	0.1	1	0	1.447	0.001	-99.967	-100.033	-200	0.049	-12.077	0.857	0.364	1.455	0.007
	0.3	0.1	1	0	1.276	0.002	-99.932	-100.068	-200	0.089	-12.037	0.875	0.424	1.272	-0.003
	0.2	0.1	1	0	0.976	-0.002	-100.052	-99.9483	-200	-0.052	-12.178	0.889	0.485	0.970	-0.007
	0.1	0.1	1	0	0.550	0.000	-100.008	-99.9918	-200	-0.005	-12.130	0.900	0.545	0.545	-0.005
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 7: Initial Step Code ($x^*=0.6$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p>
-12.28146551

			Inlet		TDMA Solution i=6		Coefficients					P and Q Terms		TDMA Solution i=7	
x^*	y^*	$u^*\Delta y$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	An	As	Ap	Bu	D	P	Q	u^*	v^*
0.7	1	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.545	0.005	-99.857	-100.143	-200	-0.083	-12.364	0.499	0.062	0.547	-0.002
	0.8	0.1	1	0	0.970	0.007	-99.795	-100.205	-200	-0.206	-12.487	0.665	0.125	0.972	-0.003
	0.7	0.1	1	0	1.272	0.003	-99.904	-100.096	-200	-0.125	-12.407	0.749	0.186	1.274	-0.001
	0.6	0.1	1	0	1.455	-0.007	-100.221	-99.7792	-200	0.329	-11.953	0.800	0.244	1.452	0.003
	0.5	0.1	1	0	1.516	0.000	-100.000	-100	-200	0.692	-11.589	0.833	0.300	1.510	0.000
	0.4	0.1	1	0	1.455	0.007	-99.779	-100.221	-200	0.329	-11.953	0.857	0.361	1.452	-0.003
	0.3	0.1	1	0	1.272	-0.003	-100.096	-99.9041	-200	-0.125	-12.407	0.875	0.423	1.274	0.001
	0.2	0.1	1	0	0.970	-0.007	-100.205	-99.7949	-200	-0.206	-12.487	0.889	0.486	0.972	0.003
	0.1	0.1	1	0	0.545	-0.005	-100.143	-99.8568	-200	-0.083	-12.364	0.900	0.547	0.547	0.002
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 8: Initial Step Code ($x^*=0.7$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p>
-12.05968738

			Inlet		TDMA Solution i=7		Coefficients					P and Q Terms		TDMA Solution i=8	
x^*	y^*	$u^*\Delta y$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	A_n	A_s	A_p	B_u	D	P	Q	u^*	v^*
0.8	1	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.547	-0.002	-100.0541	-99.9459	-200	0.031	-12.028	0.500	0.060	0.545	0.002
	0.8	0.1	1	0	0.972	-0.003	-100.0774	-99.9226	-200	0.078	-11.982	0.667	0.120	0.969	0.004
	0.7	0.1	1	0	1.274	-0.001	-100.0413	-99.9587	-200	0.054	-12.006	0.750	0.180	1.272	0.002
	0.6	0.1	1	0	1.452	0.003	-99.9179	-100.082	-200	-0.122	-12.182	0.800	0.242	1.456	-0.004
	0.5	0.1	1	0	1.510	0.000	-100.0000	-100	-200	-0.280	-12.340	0.833	0.304	1.517	0.000
	0.4	0.1	1	0	1.452	-0.003	-100.0821	-99.9179	-200	-0.122	-12.182	0.857	0.365	1.456	0.004
	0.3	0.1	1	0	1.274	0.001	-99.9587	-100.041	-200	0.054	-12.006	0.875	0.425	1.272	-0.002
	0.2	0.1	1	0	0.972	0.003	-99.9226	-100.077	-200	0.078	-11.982	0.889	0.485	0.969	-0.004
	0.1	0.1	1	0	0.547	0.002	-99.9459	-100.054	-200	0.031	-12.028	0.900	0.545	0.545	-0.002
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 9: Initial Step Code ($x^*=0.8$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p> <p style="text-align: center;">-12.20322857</p>
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			Inlet		TDMA Solution i=8		Coefficients					P and Q Terms		TDMA Solution i=9	
x^*	y^*	$u^*\text{deltay}$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	An	As	Ap	Bu	D	P	Q	u^*	v^*
0.9	1	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.545	0.002	-99.926	-100.074	-200	-0.043	-12.246	0.500	0.061	0.546	-0.002
	0.8	0.1	1	0	0.969	0.004	-99.895	-100.105	-200	-0.106	-12.309	0.666	0.123	0.971	-0.002
	0.7	0.1	1	0	1.272	0.002	-99.954	-100.046	-200	-0.060	-12.264	0.749	0.184	1.273	-0.001
	0.6	0.1	1	0	1.456	-0.004	-100.113	-99.8869	-200	0.168	-12.035	0.800	0.243	1.453	0.002
	0.5	0.1	1	0	1.517	0.000	-100.000	-100	-200	0.350	-11.853	0.833	0.301	1.512	0.000
	0.4	0.1	1	0	1.456	0.004	-99.887	-100.113	-200	0.168	-12.035	0.857	0.362	1.453	-0.002
	0.3	0.1	1	0	1.272	-0.002	-100.046	-99.9537	-200	-0.060	-12.264	0.875	0.424	1.273	0.001
	0.2	0.1	1	0	0.969	-0.004	-100.105	-99.8947	-200	-0.106	-12.309	0.889	0.485	0.971	0.002
	0.1	0.1	1	0	0.545	-0.002	-100.074	-99.9257	-200	-0.043	-12.246	0.900	0.546	0.546	0.002
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 10: Initial Step Code ($x^*=0.9$)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p>
-12.06850809

			Inlet		TDMA Solution i=9		Coefficients					P and Q Terms		TDMA Solution i=10	
x*	y*	u* Δt	u*(x=0)	v*(x=0)	u*	v*	An	As	Ap	Bu	D	P	Q	u*	v*
1	1	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.546	-0.002	-100.0480	-99.952	-200	0.028	-12.041	0.500	0.060	0.545	0.001
	0.8	0.1	1	0	0.971	-0.002	-100.0681	-99.9319	-200	0.068	-12.000	0.667	0.120	0.969	0.002
	0.7	0.1	1	0	1.273	-0.001	-100.0299	-99.9701	-200	0.039	-12.029	0.750	0.180	1.272	0.001
	0.6	0.1	1	0	1.453	0.002	-99.9270	-100.073	-200	-0.108	-12.177	0.800	0.242	1.455	-0.002
	0.5	0.1	1	0	1.512	0.000	-100.0000	-100	-200	-0.226	-12.294	0.833	0.304	1.517	0.000
	0.4	0.1	1	0	1.453	-0.002	-100.0730	-99.927	-200	-0.108	-12.177	0.857	0.365	1.455	0.002
	0.3	0.1	1	0	1.273	0.001	-99.9701	-100.03	-200	0.039	-12.029	0.875	0.425	1.272	-0.001
	0.2	0.1	1	0	0.971	0.002	-99.9319	-100.068	-200	0.068	-12.000	0.889	0.485	0.969	-0.002
	0.1	0.1	1	0	0.546	0.002	-99.9520	-100.048	-200	0.028	-12.041	0.900	0.545	0.545	-0.001
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 11: Initial Step Code (x*=1.0)

<p style="text-align: center;">dp/dx Iteration for Mass Conservation, C</p> <p style="text-align: center;">-12.17031502</p>
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			Inlet		TDMA Solution i=10		Coefficients					P and Q Terms		TDMA Solution i=11	
x^*	y^*	$u^*\text{deltay}$	$u^*(x=0)$	$v^*(x=0)$	u^*	v^*	A_n	A_s	A_p	B_u	D	P	Q	u^*	v^*
1.1	1	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	0.9	0.1	1	0	0.545	0.001	-99.9553	-100.045	-200	-0.03	-12.20	0.50	0.06	0.546	-0.001
	0.8	0.1	1	0	0.969	0.002	-99.9367	-100.063	-200	-0.06	-12.23	0.67	0.12	0.970	-0.002
	0.7	0.1	1	0	1.272	0.001	-99.9728	-100.027	-200	-0.04	-12.21	0.75	0.18	1.273	-0.001
	0.6	0.1	1	0	1.455	-0.002	-100.0681	-99.9319	-200	0.10	-12.07	0.80	0.24	1.454	0.002
	0.5	0.1	1	0	1.517	0.000	-100.0000	-100	-200	0.21	-11.96	0.83	0.30	1.514	0.000
	0.4	0.1	1	0	1.455	0.002	-99.9319	-100.068	-200	0.10	-12.07	0.86	0.36	1.454	-0.002
	0.3	0.1	1	0	1.272	-0.001	-100.0272	-99.9728	-200	-0.04	-12.21	0.87	0.42	1.273	0.001
	0.2	0.1	1	0	0.969	-0.002	-100.0633	-99.9367	-200	-0.06	-12.23	0.89	0.49	0.970	0.002
	0.1	0.1	1	0	0.545	-0.001	-100.0447	-99.9553	-200	-0.03	-12.20	0.90	0.55	0.546	0.001
	0	0.1	1	0	0.000	0.000	0	0	1	0	0	0	0	0	0.000
	Mass=	1											Mass=	1	

Table 12: Initial Step Code ($x^*=1.1$)

As seen, the pressure gradient starts off with a -22.823 and gradually is converging to -12. Consequently, the velocity at the center of the channel in the x direction (u^*) also starts off around 1.29 and gradually is getting to the true value of 1.5. The flow iterations, the pressure gradient convergence table, and convergence graph can be seen below.

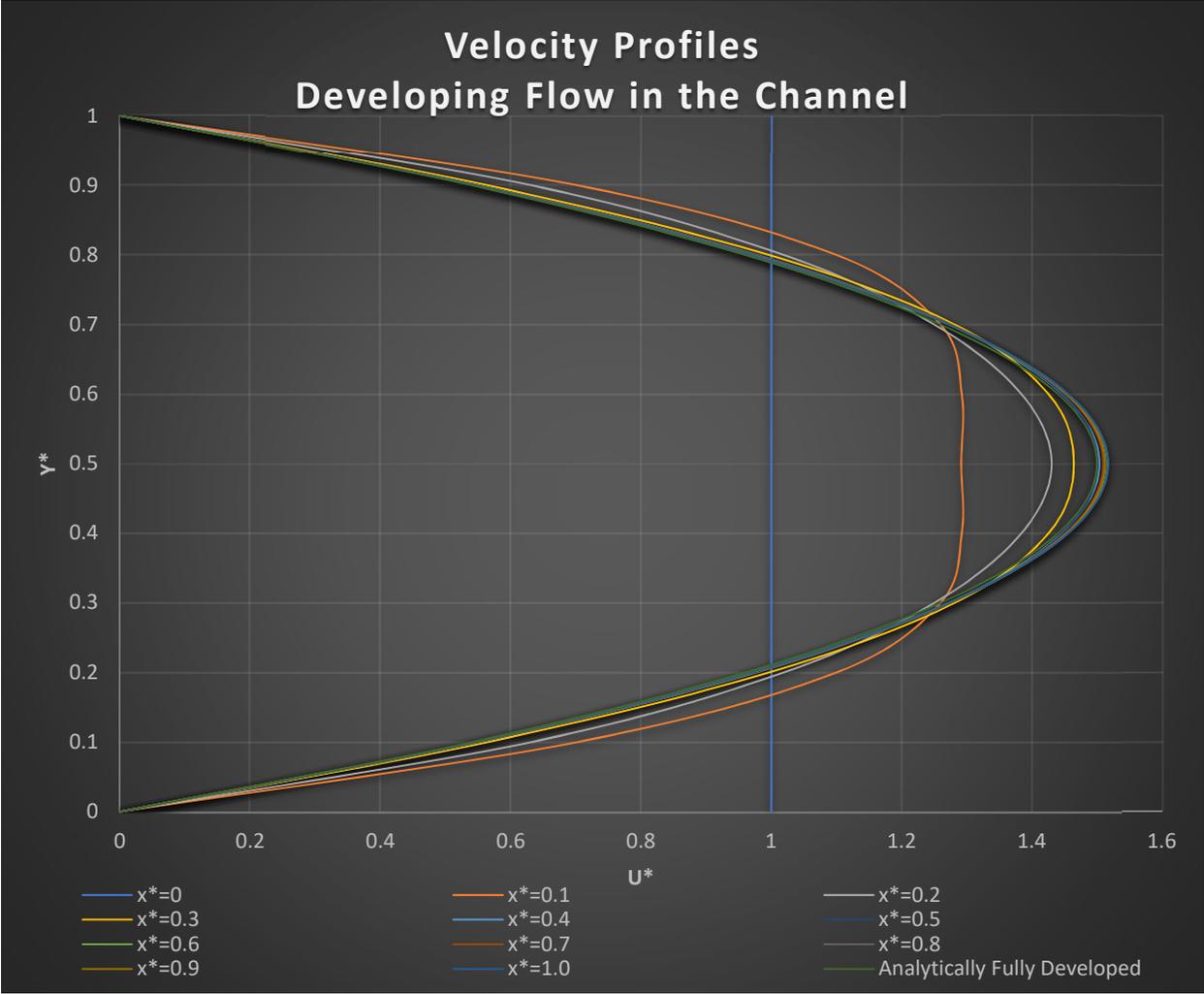


Figure 8: Developing Flow in a Channel Velocity Profiles

x^*	dp/dx
0	-22.8231
0.1	-20.8963
0.2	-13.8887
0.3	-13.0461
0.4	-12.5543
0.5	-12.1257
0.6	-12.2815
0.7	-12.0597
0.8	-12.2032
0.9	-12.0685
1	-12.1703

Table 13: Pressure Gradient Convergence

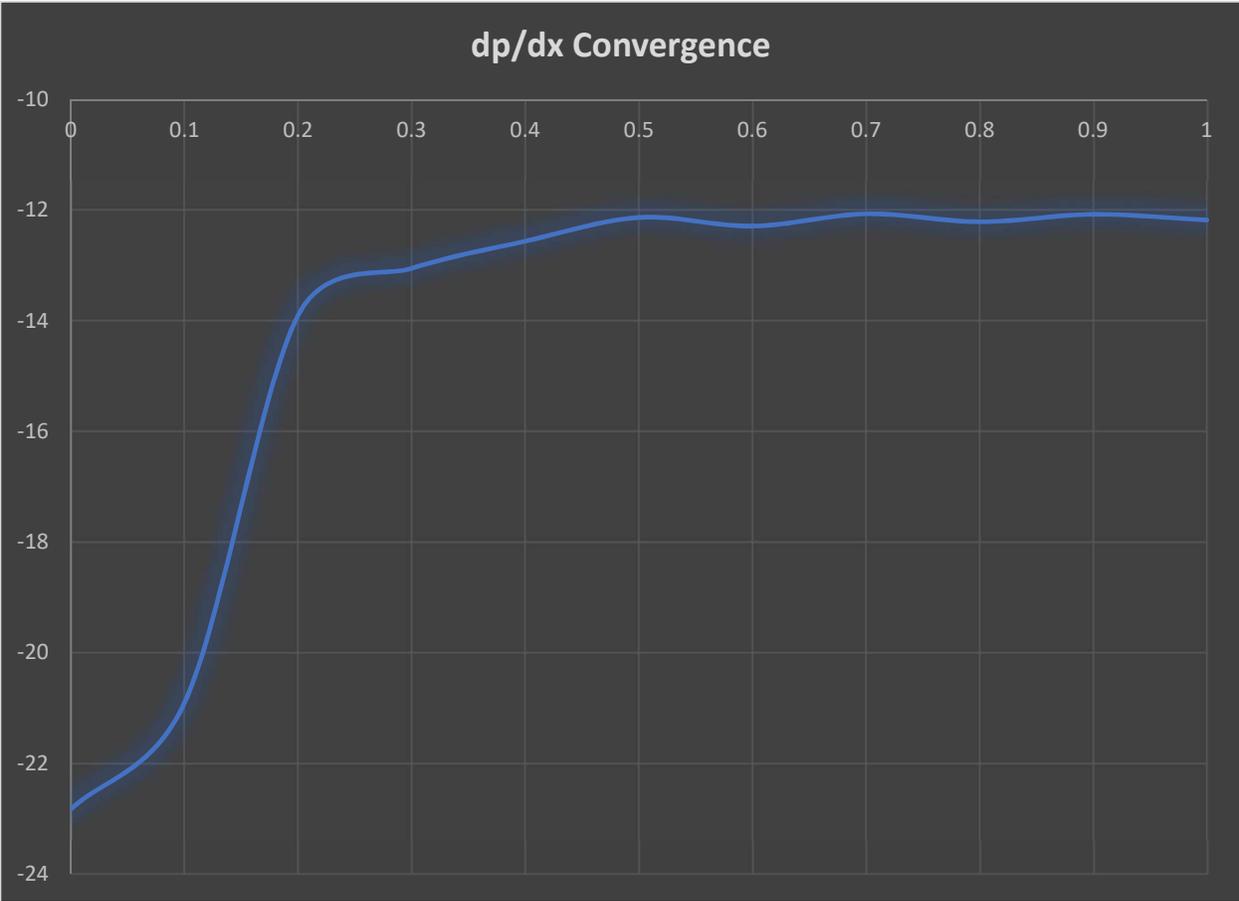


Figure 9: dp/dx , Pressure Gradient Convergence

As with all analysis, we must compare our results with other methods to ensure we are correct in our work. A couple of methods for comparison include the boundary layer polynomial approximation and the commencement flow in a circular pipe. Both methods are appropriate in verifying our method though the commencement flow in a circular pipe is very complex and will also result in a slightly different solution since the flow is through a circular pipe instead of the channel. Below is the verification using the boundary layer polynomial approximation.

First, we start with the boundary layer representation that was given in the problem statement.

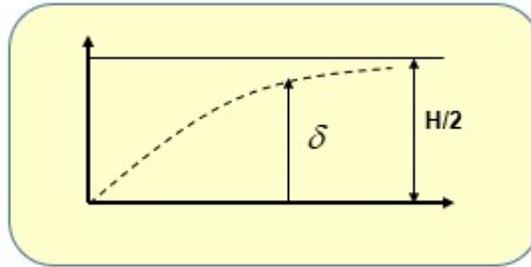


Figure 10: Boundary Layer Representation

We first fit a polynomial that can approximately represent the PDE as

$$u = a + by + cy^2 \quad (1.57)$$

Since we can approximate the shape to be second order. Now to find the slope, we take the partial derivative which yields

$$\frac{\partial u}{\partial y} = b + 2cy \quad (1.58)$$

$$\left. \frac{\partial u}{\partial y} \right|_{y=0} = \left. \left(\frac{\partial u}{\partial y} \right)_1 = b \quad (1.59)$$

Obtaining coefficients that represent the velocities as we move along the y axis with constant deltas can be represented from the Taylor series by

$$u_1 = a \quad (1.60)$$

$$u_2 = a + b\Delta y + c\Delta y^2 \quad (1.61)$$

$$u_3 = a + b(2\Delta y) + c(2\Delta y)^2 \quad (1.62)$$

$$b = \frac{-3u_1 + 4u_2 - u_3}{2\Delta y} \quad (1.63)$$

$$\left. \left(\frac{\partial u}{\partial y} \right)_1 = \frac{-3u_1 + 4u_2 - u_3}{2\Delta y} + O(\Delta y)^2 \quad (1.64)$$

$$\tau = \mu \left. \left(\frac{\partial u}{\partial y} \right)_1 \quad (1.65)$$

This method for our purposes can be solved the same way, but using dimensionless for and transforming to the computational domain

$$u^* = a(x^*) + b(x^*)\eta + c(x^*)\eta^2 \quad (1.66)$$

where $\eta = y/\delta$.

We know that due to our boundary conditions, at $y = 0$, $u^* = 0$, which yields a $\eta = 0$. When $y = \delta$, plugging into $\eta = y/\delta$, $\eta = 1$ which yields $u^* = 1$. And due to the solving of the polynomial fit the can then say that

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right) = 0 \quad (1.67)$$

Therefore

$$\tau = \mu \left(\frac{\partial u^*}{\partial \eta} \right) = 0 \quad (1.68)$$

This leads to

$$u^* = \frac{u}{U} = 2\eta - \eta^2 \quad (1.69)$$

To evaluate for the fully developed flow, we know that the integral of all of the velocities under the boundary plus the difference of half the channel and the boundary layer times the velocity U will be equal to the inlet velocity times half of the channel being evaluated

$$U_i \left(\frac{H}{2} \right) = \int_0^\delta u dy + \left(\frac{H}{2} - \delta \right) U \quad (1.70)$$

$$U_i \left(\frac{H}{2} \right) = U\delta \left(\frac{2}{3} \right) + \left(\frac{H}{2} - \delta \right) U \quad (1.71)$$

$$\delta = \frac{3}{2}H \left(1 - \frac{U_i}{U} \right) \quad (1.72)$$

Equation 1.72 allows us to solve for U/U_i at any location in the channel of the boundary layer. The location of interest is when $\delta = H/2$, which yields

$$\frac{1}{2}H = \frac{3}{2}H \left(1 - \frac{U_i}{U} \right) \quad (1.73)$$

$$\frac{1}{1 - \left(\frac{1}{3} \right)} = \frac{U}{U_i} = 1.5 \quad (1.74)$$

Discussion

The solutions within this report should be considered only an approximation to the result. We know that discretization methods that transform the partial differential equations that govern the fluid behavior through the domain into algebraic form are not exact numerical values. The finite difference scheme evaluates the dependent variable at discrete points in the domain. This method was used due to its inherent ability to be easily coded, though it may not be the most accurate method. We must consider errors such as the truncation error and round off error when approaching this problem, which is why we do not see the exact pressure gradient convergence or velocity at the center of the channel. We could also increase the number of nodes and elements in the grid generation, creating a finer mesh, which would allow us to continue to iterate on the pressure gradient. The more we can iterate, the closer we can come to a full convergence because the previous iterations are used to formulate the next step due to the upwind scheme that moves the primary node to the node to the right or left. This allows us to put different weight into different coefficients that give us more accurate results.

This problem lends itself well to the finite difference scheme due to the simplicity of the physical domain. It should also be noted that due to the many assumptions, this problem was able to be represented in simplified forms of the continuity and momentum equations while also excluding the energy equation. This is all due to the steady state, two-dimensional, and incompressibility of the fluid flow, as well as the given boundary conditions. With steady state condition, we do not analyze the relation of time to the flow. A two-dimensional problem allows the continuity equation and momentum equations to be reduced to only the x and y components. Not all problems dealing with a flow through a channel will use the same approach. There may be considerations for an unsteady flow that is compressible in three dimensions. We could also have a non-uniform velocity profile entering the pipe or a complex wall geometry that may lend itself better to another approach such as a finite element method or finite volume method.

Analysis of any fluid dynamic problem involves the use of the conservation laws, which were used extensively in this solution. Obtaining the governing equations for the problem allow us to study the behavior of the channel. From the continuity equation, momentum equation, and our boundary conditions, we can obtain the governing equations to fully describe our problem. Since these are very difficult to solve, we use the discretization method of finite difference to transform the PDE into algebraic form. This makes the solving easy to compute. Also, to ease in the calculation, non-dimensionalized forms of our variables and governing equations can be obtained which scales all the values in relation to a unit, also called scaling or normalizing. This can be simplified even further into calculated coefficients that will make up the computational molecule.

An extremely important method for solving this problem is the TDMA algorithm. This method is an iterative 1D method that uses recurrence terms to allow us to solve for u^* at all steps. TDMA takes a mesh and breaks it down even further for ease of calculation. We can then back-substitute to obtain the local velocities. We then iterate the pressure gradient to convergence, and we can satisfy our global continuity. We then use the velocity components as the upstream conditions for the next TDMA step.

Verification of methods is a vital part in numerical solutions. These methods provide fairly accurate solutions quickly, but we need to check if our solution is accurate enough. We can our finite difference approach with the polynomial fitting of the boundary layer. We found that both the finite difference discretization and the polynomial approximation fitting both yield a velocity

of 1.5 at the center of the fully developed flow channel. We can also use other methods such as commencement flow through a circular pipe which will be different than the channel since we will have three dimensions as well as polar coordinates.

Future work can be conducted on such a problem, checking different methods for accuracy and errors. The finite element method using weighted residuals or variational approach could be used and the finite volume method could also be used. These two methods would be better to use but would take much more time to compute. Overall, the geometry of this problem is simple and since we have a variety of conditions that simplify this problem, the finite difference method is a very fair approach to evaluating the governing equations obtained.

Appendix

Sample Calculation Code for First Three Steps

delta x		delta y		Re	Alpha	dp/dx Iteration for Mass Conservation, C		G		I		J		K		L		M		N		O		P		Q		R	
0.1	0.1	0.1	0.1	30	0.1	-22.8231246267338																							
<p>dp/dx Iteration for Mass Conservation, C</p> <p>-22.8231246267338</p>																													
Inlet		Closed Form with dp/dx = -12		Guess		Coefficients										P and Q Terms				TDMA Solution i=1									
x*	y*	u*delta y	u*(z=0)	v*(z=0)	u*(z=0.1)	v*(z=0.1)	Aa	As	Ap	Ba	Bs	Bt	D	P	Q	u*	v*												
1	0.1	0.1	1	0	=0.5*-12*(D9^2)-D9	0	0	0	1	=((E13*F13^2)/W11)*((W11-1)/C13)-((F13^2)/((W11-1)*(W11-1)-(2*W11)))/(C13^2)	0	0	=J9/(L9-K9*O)	=((N9-K9*O)/L9-K9*O)	=((N9-K9*O)/L9-K9*O)	0	0												
0.9	0.1	1	1	0	=0.5*-12*(D10^2)-D10	0.01	=-(W4D13^2)*((E13*F13^110)/4D13)	=-(W4D13^2)*((E13*F13^110)/4D13)	=J10+K10	=((E13*F13^110)/(H10-F10)/C13)-((F13^2)/((F10+H10)-(2*H10)))/(C13^2)	=M10+H13	=J10/(L10-K10*O9)	=((N10-K10*P9)/(L10-K10*O9))	=((N10-K10*P9)/(L10-K10*O9))	=((O10*Q11)+P10)	=((O10*Q11)+P10)	0.1												
0.8	0.1	1	1	0	=0.5*-12*(D11^2)-D11	0.01	=-(W4D13^2)*((E13*F13^111)/4D13)	=-(W4D13^2)*((E13*F13^111)/4D13)	=J11+K11	=((E13*F13^111)/(H11-F11)/C13)-((F13^2)/((F11+H11)-(2*H11)))/(C13^2)	=M11+H13	=J11/(L11-K11*O10)	=((N11-K11*P10)/(L11-K11*O10))	=((N11-K11*P10)/(L11-K11*O10))	=((O11*Q12)+P11)	=((O11*Q12)+P11)	0.1												
0.7	0.1	1	1	0	=0.5*-12*(D12^2)-D12	0.01	=-(W4D13^2)*((E13*F13^112)/4D13)	=-(W4D13^2)*((E13*F13^112)/4D13)	=J12+K12	=((E13*F13^112)/(H12-F12)/C13)-((F13^2)/((F12+H12)-(2*H12)))/(C13^2)	=M12+H13	=J12/(L12-K12*O11)	=((N12-K12*P11)/(L12-K12*O11))	=((N12-K12*P11)/(L12-K12*O11))	=((O12*Q13)+P12)	=((O12*Q13)+P12)	0.1												
0.6	0.1	1	1	0	=0.5*-12*(D13^2)-D13	0.01	=-(W4D13^2)*((E13*F13^113)/4D13)	=-(W4D13^2)*((E13*F13^113)/4D13)	=J13+K13	=((E13*F13^113)/(H13-F13)/C13)-((F13^2)/((F13+H13)-(2*H13)))/(C13^2)	=M13+H13	=J13/(L13-K13*O12)	=((N13-K13*P12)/(L13-K13*O12))	=((N13-K13*P12)/(L13-K13*O12))	=((O13*Q14)+P13)	=((O13*Q14)+P13)	0.1												
0.5	0.1	1	1	0	=0.5*-12*(D14^2)-D14	0	=-(W4D13^2)*((E13*F13^114)/4D13)	=-(W4D13^2)*((E13*F13^114)/4D13)	=J14+K14	=((E13*F13^114)/(H14-F14)/C13)-((F13^2)/((F14+H14)-(2*H14)))/(C13^2)	=M14+H13	=J14/(L14-K14*O13)	=((N14-K14*P13)/(L14-K14*O13))	=((N14-K14*P13)/(L14-K14*O13))	=((O14*Q15)+P14)	=((O14*Q15)+P14)	0												
0.4	0.1	1	1	0	=0.5*-12*(D15^2)-D15	-0.01	=-(W4D13^2)*((E13*F13^115)/4D13)	=-(W4D13^2)*((E13*F13^115)/4D13)	=J15+K15	=((E13*F13^115)/(H15-F15)/C13)-((F13^2)/((F15+H15)-(2*H15)))/(C13^2)	=M15+H13	=J15/(L15-K15*O14)	=((N15-K15*P14)/(L15-K15*O14))	=((N15-K15*P14)/(L15-K15*O14))	=((O15*Q16)+P15)	=((O15*Q16)+P15)	-0.1												
0.3	0.1	1	1	0	=0.5*-12*(D16^2)-D16	-0.01	=-(W4D13^2)*((E13*F13^116)/4D13)	=-(W4D13^2)*((E13*F13^116)/4D13)	=J16+K16	=((E13*F13^116)/(H16-F16)/C13)-((F13^2)/((F16+H16)-(2*H16)))/(C13^2)	=M16+H13	=J16/(L16-K16*O15)	=((N16-K16*P15)/(L16-K16*O15))	=((N16-K16*P15)/(L16-K16*O15))	=((O16*Q17)+P16)	=((O16*Q17)+P16)	-0.1												
0.2	0.1	1	1	0	=0.5*-12*(D17^2)-D17	-0.01	=-(W4D13^2)*((E13*F13^117)/4D13)	=-(W4D13^2)*((E13*F13^117)/4D13)	=J17+K17	=((E13*F13^117)/(H17-F17)/C13)-((F13^2)/((F17+H17)-(2*H17)))/(C13^2)	=M17+H13	=J17/(L17-K17*O16)	=((N17-K17*P16)/(L17-K17*O16))	=((N17-K17*P16)/(L17-K17*O16))	=((O17*Q18)+P17)	=((O17*Q18)+P17)	-0.1												
0.1	0.1	1	1	0	=0.5*-12*(D18^2)-D18	-0.01	=-(W4D13^2)*((E13*F13^118)/4D13)	=-(W4D13^2)*((E13*F13^118)/4D13)	=J18+K18	=((E13*F13^118)/(H18-F18)/C13)-((F13^2)/((F18+H18)-(2*H18)))/(C13^2)	=M18+H13	=J18/(L18-K18*O17)	=((N18-K18*P17)/(L18-K18*O17))	=((N18-K18*P17)/(L18-K18*O17))	=((O18*Q19)+P18)	=((O18*Q19)+P18)	-0.1												
0.1	0	0	1	0	=0.5*-12*(D19^2)-D19	0	0	0	1	=((E13*F13^119)/(H19-F19)/C13)-((F13^2)/((F19+H19)-(2*H19)))/(C13^2)	0	0	=J19/(L19-K19*O18)	=((N19-K19*P18)/(L19-K19*O18))	=((N19-K19*P18)/(L19-K19*O18))	=((O19*Q20)+P19)	=((O19*Q20)+P19)	0											
Mass		=SUM(E9:E18)														Mass=7*u*Delta y		=SUM(Q9:Q19)											
<p>dp/dx Iteration for Mass Conservation, C</p> <p>-20.8362887315858</p>																													
Inlet		TDMA Solution i=1				Coefficients										P and Q Terms				TDMA Solution i=2									
x*	y*	u*delta y	u*(z=0)	v*(z=0)	u*	v*	Aa	As	Ap	Ba	Bs	Bt	D	P	Q	u*	v*												
1	0.1	0.1	1	0	0	0	=-(W4D13^2)*((E13*F13^131)/4D13)	=-(W4D13^2)*((E13*F13^131)/4D13)	=J31+K31	=((E13*F13^131)/(H31-F13)/C13)-((F13^2)/((F31+H31)-(2*H31)))/(C13^2)	0	0	=J30/(L30-K30*O)	=((N30-K30*O)/L30-K30*O)	=((N30-K30*O)/L30-K30*O)	0	0												
0.9	0.1	1	1	0	0.70207499182129	0.1	=-(W4D13^2)*((E13*F13^132)/4D13)	=-(W4D13^2)*((E13*F13^132)/4D13)	=J32+K32	=((E13*F13^132)/(H32-F13)/C13)-((F13^2)/((F32+H32)-(2*H32)))/(C13^2)	=M31+H13	=J31/(L31-K31*O3)	=((N31-K31*P30)/(L31-K31*O3))	=((N31-K31*P30)/(L31-K31*O3))	=((O31*Q32)+P31)	=((O31*Q32)+P31)	-0.1												
0.8	0.1	1	1	0	1.10003903447807	0.1	=-(W4D13^2)*((E13*F13^133)/4D13)	=-(W4D13^2)*((E13*F13^133)/4D13)	=J33+K33	=((E13*F13^133)/(H33-F13)/C13)-((F13^2)/((F33+H33)-(2*H33)))/(C13^2)	=M32+H13	=J32/(L32-K32*O4)	=((N32-K32*P31)/(L32-K32*O4))	=((N32-K32*P31)/(L32-K32*O4))	=((O32*Q33)+P32)	=((O32*Q33)+P32)	-0.1												
0.7	0.1	1	1	0	1.25964453950999	0.1	=-(W4D13^2)*((E13*F13^134)/4D13)	=-(W4D13^2)*((E13*F13^134)/4D13)	=J34+K34	=((E13*F13^134)/(H34-F13)/C13)-((F13^2)/((F34+H34)-(2*H34)))/(C13^2)	=M33+H13	=J33/(L33-K33*O5)	=((N33-K33*P32)/(L33-K33*O5))	=((N33-K33*P32)/(L33-K33*O5))	=((O33*Q34)+P33)	=((O33*Q34)+P33)	-0.1												
0.6	0.1	1	1	0	1.29244574841578	0.1	=-(W4D13^2)*((E13*F13^135)/4D13)	=-(W4D13^2)*((E13*F13^135)/4D13)	=J35+K35	=((E13*F13^135)/(H35-F13)/C13)-((F13^2)/((F35+H35)-(2*H35)))/(C13^2)	=M34+H13	=J34/(L34-K34*O6)	=((N34-K34*P33)/(L34-K34*O6))	=((N34-K34*P33)/(L34-K34*O6))	=((O34*Q35)+P34)	=((O34*Q35)+P34)	-0.1												
0.5	0.1	1	1	0	1.29153137154975	0	=-(W4D13^2)*((E13*F13^136)/4D13)	=-(W4D13^2)*((E13*F13^136)/4D13)	=J36+K36	=((E13*F13^136)/(H36-F13)/C13)-((F13^2)/((F36+H36)-(2*H36)))/(C13^2)	=M35+H13	=J35/(L35-K35*O7)	=((N35-K35*P34)/(L35-K35*O7))	=((N35-K35*P34)/(L35-K35*O7))	=((O35*Q36)+P35)	=((O35*Q36)+P35)	0												
0.4	0.1	1	1	0	1.29245174841578	-0.1	=-(W4D13^2)*((E13*F13^137)/4D13)	=-(W4D13^2)*((E13*F13^137)/4D13)	=J37+K37	=((E13*F13^137)/(H37-F13)/C13)-((F13^2)/((F37+H37)-(2*H37)))/(C13^2)	=M36+H13	=J36/(L36-K36*O8)	=((N36-K36*P35)/(L36-K36*O8))	=((N36-K36*P35)/(L36-K36*O8))	=((O36*Q37)+P36)	=((O36*Q37)+P36)	-0.1												
0.3	0.1	1	1	0	1.25964453950999	-0.1	=-(W4D13^2)*((E13*F13^138)/4D13)	=-(W4D13^2)*((E13*F13^138)/4D13)	=J38+K38	=((E13*F13^138)/(H38-F13)/C13)-((F13^2)/((F38+H38)-(2*H38)))/(C13^2)	=M37+H13	=J37/(L37-K37*O9)	=((N37-K37*P36)/(L37-K37*O9))	=((N37-K37*P36)/(L37-K37*O9))	=((O37*Q38)+P37)	=((O37*Q38)+P37)	-0.1												
0.2	0.1	1	1	0	1.10003903447807	-0.1	=-(W4D13^2)*((E13*F13^139)/4D13)	=-(W4D13^2)*((E13*F13^139)/4D13)	=J39+K39	=((E13*F13^139)/(H39-F13)/C13)-((F13^2)/((F39+H39)-(2*H39)))/(C13^2)	=M38+H13	=J38/(L38-K38*O10)	=((N38-K38*P37)/(L38-K38*O10))	=((N38-K38*P37)/(L38-K38*O10))	=((O38*Q39)+P38)	=((O38*Q39)+P38)	-0.1												
0.1	0.1	1	1	0	0.702074991821289	-0.1	=-(W4D13^2)*((E13*F13^140)/4D13)	=-(W4D13^2)*((E13*F13^140)/4D13)	=J40+K40	=((E13*F13^140)/(H40-F13)/C13)-((F13^2)/((F40+H40)-(2*H40)))/(C13^2)	=M39+H13	=J39/(L39-K39*O11)	=((N39-K39*P38)/(L39-K39*O11))	=((N39-K39*P38)/(L39-K39*O11))	=((O39*Q40)+P39)	=((O39*Q40)+P39)	-0.1												
0.2	0.1	1	1	0	0	0	0	0	1	=((E13*F13^140)/(H40-F13)/C13)-((F13^2)/((F40+H40)-(2*H40)))/(C13^2)	0	0	=J40/(L40-K40*O12)	=((N40-K40*P39)/(L40-K40*O12))	=((N40-K40*P39)/(L40-K40*O12))	=((O40*Q41)+P40)	=((O40*Q41)+P40)	-0.1											
Mass		=SUM(E30:E39)														Mass=7*u*Delta y		=SUM(Q30:Q41)											
<p>dp/dx Iteration for Mass Conservation, C</p> <p>-13.888658527529</p>																													
Inlet		TDMA Solution i=2				Coefficients										P and Q Terms				TDMA Solution i=3									
x*	y*	u*delta y	u*(z=0)	v*(z=0)	u*	v*	Aa	As	Ap	Ba	Bs	Bt	D	P	Q	u*	v*												
1	0.1	0.1	1	0	0	0	=-(W4D13^2)*((E13*F13^153)/4D13)	=-(W4D13^2)*((E13*F13^153)/4D13)	=J53+K53	=((E13*F13^153)/(H53-F13)/C13)-((F13^2)/((F53+H53)-(2*H53)))/(C13^2)	0	0	=J52/(L52-K52*O)	=((N52-K52*O)/L52-K52*O)	=((N52-K52*O)/L52-K52*O)	0	0												
0.9	0.1	1	1	0	0.631588154214064	0.070486837607	=-(W4D13^2)*((E13*F13^154)/4D13)	=-(W4D13^2)*((E13*F13^154)/4D13)	=J54+K54	=((E13*F13^154)/(H54-F13)/C13)-((F13^2)/((F54+H54)-(2*H54)))/(C13^2)	=M53+H13	=J53/(L53-K53*O5)	=((N53-K53*P52)/(L53-K53*O5))	=((N53-K53*P52)/(L53-K53*O5))	=((O53*Q54)+P53)	=((O53*Q54)+P53)	-0.1												
0.8	0.1	1	1	0	1.01905614610364	0.081042888368	=-(W4D13^2)*((E13*F13^155)/4D13)	=-(W4D13^2)*((E13*F13^155)/4D13)	=J55+K55	=((E13*F13^155)/(H55-F13)/C13)-((F13^2)/((F55+H55)-(2*H55)))/(C13^2)	=M54+H13	=J54/(L54-K54*O6)	=((N54-K54*P53)/(L54-K54*O6))	=((N54-K54*P53)/(L54-K54*O6))	=((O54*Q55)+P54)	=((O54*Q55)+P54)	-0.1												
0.7	0.1	1	1	0	1.2501548987239	0.009489640786	=-(W4D13^2)*((E13*F13^156)/4D13)	=-(W4D13^2)*((E13*F13^156)/4D13)	=J56+K56	=((E13*F13^156)/(H56-F13)/C13)-((F13^2)/((F56+H56)-(2*H56)))/(C13^2)	=M55+H13	=J55/(L55-K55*O7)	=((N55-K55*P54)/(L55-K55*O7))	=((N55-K55*P54)/(L55-K55*O7))	=((O55*Q56)+P55)	=((O55*Q56)+P55)	-0.1												
0.6	0.1	1	1	0	1.38335203397557	-0.0915362855591	=-(W4D13^2)*((E13*F13^157)/4D13)	=-(W4D13^2)*((E13*F13^157)/4D13)	=J57+K57	=((E13*F13^157)/(H57-F13)/C13)-((F13^2)/((F57+H57)-(2*H57)))/(C13^2)	=M56+H13	=J56/(L56-K56*O8)	=((N56-K56*P55)/(L56-K56*O8))	=((N56-K56*P55)/(L56-K56*O8))	=((O56*Q57)+P56)	=((O56*Q57)+P56)	-0.1												
0.5	0.1	1	1	0	1.43049753395364	0	=-(W4D13^2)*((E13*F13^158)/4D13)	=-(W4D13^2)*((E13*F13^158)/4D13)	=J58+K58	=((E13*F13^158)/(H58-F13)/C13)-((F13^2)/((F58+H58)-(2*H58)))/(C13^2)	=M57+H13	=J57/(L57-K57*O9)	=((N57-K57*P56)/(L57-K57*O9))	=((N57-K57*P56)/(L57-K57*O9))	=((O57*Q58)+P57)	=((O57*Q58)+P57)	0												
0.4	0.1	1	1	0	1.38335203397557	0.0915362855591	=-(W4D13^2)*((E13*F13^159)/4D13)	=-(W4D13^2)*((E13*F13^159)/4D13)	=J59+K59	=((E13*F13^159)/(H59-F13)/C13)-((F13^2)/((F59+H59)-(2*H59)))/(C13^2)	=M58+H13	=J58/(L58-K58*O10)	=((N58-K58*P57)/(L58-K58*O10))	=((N58-K58*P57)/(L58-K58*O10))	=((O58*Q59)+P58)	=((O58*Q59)+P58)	-0.1												
0.3	0.1	1	1	0	1.2501548987239	-0.009489640786	=-(W4D13^2)*((E13*F13^160)/4D13)	=-(W4D13^2)*((E13*F13^160)/4D13)	=J60+K60	=((E13*F13^160)/(H60-F13)/C13)-((F13^2)/((F60+H60)-(2*H60)))/(C13^2)	=M59+H13	=J59/(L59-K59*O11)	=((N59-K59*P58)/(L59-K59*O11))	=((N59-K59*P58)/(L59-K59*O11))	=((O59*Q60)+P59)	=((O59*Q60)+P59)	-0.1												
0.2	0.1	1	1	0	1.01905614610364	-0.081042888368	=-(W4D13^2)*((E13*F13^161)/4D13)	=-(W4D13^2)*((E13*F13^161)/4D13)	=J61+K61	=((E13*F13^161)/(H61-F13)/C13)-((F13^2)/((F61+H61)-(2*H61)))/(C13^2)	=M60+H13	=J60/(L60-K60*O12)	=((N60-K60*P59)/(L60-K60*O12))	=((N60-K60*P59)/(L60-K60*O12))	=((O60*Q61)+P60)	=((O60*Q61)+P60)	-0.1												
0.1	0.1	1	1	0	0.631588154214064	-0.070486837607	=-(W4D13^2)*((E13*F13^162)/4D13)	=-(W4D13^2)*((E13*F13^162)/4D13)	=J62+K62	=((E13*F13^162)/(H62-F13)/C13)-((F13^2)/((F62+H62)-(2*H62)))/(C13^2)	=M61+H13	=J61/(L61-K61*O13)	=((N61-K61*P60)/(L61-K61*O13))	=((N61-K61*P60)/(L61-K61*O13))	=														

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