Rensselaer Polytechnic Institute

MANE 6720: Computational Fluid Dynamics

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Final Project
CFD for Nozzle Flow

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List of Symbols

α	Relaxation Factor
A_A	Cross-Sectional Area at A
A_B	Cross-Sectional Area at B
A	Point A in Nozzle
B	Point B in Nozzle
e	Intermediate Node on Staggered Grid
F	Flow Rate
g_x	Gravitational Force
∞	Infinity
n	Nodal Location
p	Pressure
p^*	Imperfect Pressure
p'	Pressure Correction
R	Residual
ρ	Density
u	Velocity (x-direction)
u^*	Imperfect Velocity (x-direction)
u'	Velocity Correction (x-direction)
v	Velocity (y-direction)
v^*	Imperfect Velocity (y-direction)
v'	Velocity Correction (y-direction)
V	Volume
w	Velocity (z-direction)
w^*	Imperfect Velocity (z-direction)
w'	Velocity Correction (z-direction)

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Abstract

A one-dimensional, steady state, incompressible flow with negligible momentum upstream of point one before a nozzle inlet is to be analyzed to study the pressure at the center as well as the velocities throughout the nozzle. Utilizing given boundary conditions, equations to describe the flow nozzle, initial approximations, and governing equations of fluid dynamics we are able to obtain the governing equations and discretization equations to describe the problem at hand. Further, using the iterative solution process in the SIMPLE algorithm and studying the effects of relaxation methods, under relaxation values of $\alpha_u = 0.6$ and $\alpha_p = 0.38$ were found to offer the best rate of convergence and stability to obtain the velocities at section A and B to be $u_A = 2$ and $u_B = 6$ and the pressure at point two to be $p_2 = 24$.

Introduction/Assumptions

The problem at hand consists of a one-dimensional nozzle as shown in Figure 1 below. Formulation of discretization equations for u and p' are to be obtained and then used to acquire the values of the velocity at location A and B and also the pressure at point 2 in the nozzle.

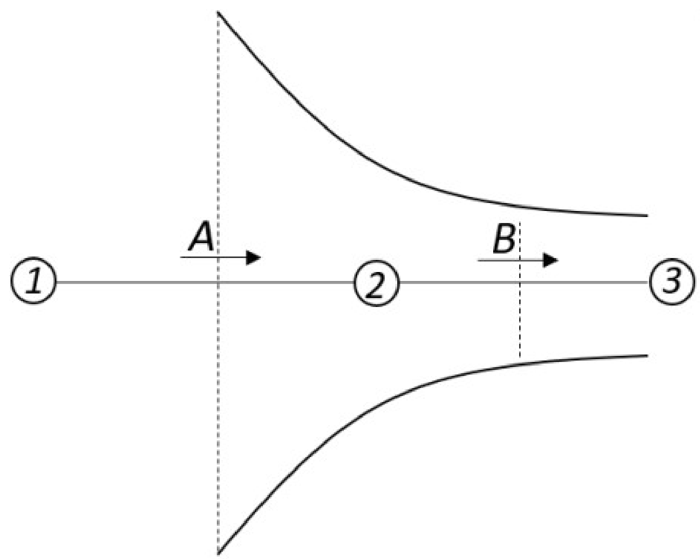

Figure 1: Flow Nozzle

To accurately solve this problem, we must consider a few assumptions. As stated above, we are assuming a one-dimensional nozzle and the fluid upstream of point one has negligible momentum. We also assume that the fluid in the nozzle is steady and incompressible, we are dealing with a closed system, and the effects of gravity are negligible. We also will not consider the Energy Equation since we do not have any temperature or energy loss or generation in this problem.

The fluid in the nozzle is of constant density, $\rho = 1$. The inlet pressure to the nozzle at point one is $p_1 = 28$ and the outlet pressure at point three is $p_3 = 0$. The cross-sectional area, A, at section A in the nozzle is $A_A = 3$ and at section B in the nozzle is $A_B = 1$.

It has been given that the one-dimensional flow in the nozzle is described by

$$\frac{d}{dx}(\rho A u) = 0 \tag{1.0}$$

$$\frac{d}{dx}(\rho A u)u = -A\frac{dp}{dx} \tag{1.1}$$

To solve this problem with convergence and stability, we will utilize the iterative SIMPLE algorithm which will start with an initial approximation of the pressure field which will be used in the momentum equation to find the velocities at specified points in the nozzle. We will then use these velocities and approximated pressure in an effort to satisfy the continuity equation. If the continuity equation is satisfied, then we have approximated the proper pressure field. If the continuity equation is not satisfied, then we will have a mass residual and will need to define a pressure correction to update our approximated pressure. We will then start the process over again until the continuity equation is satisfied. We are given an initial guess of F or $\rho u A$ to be 5.

Conclusions

The purpose of this report is to analyze the one-dimensional flow in a nozzle. Through our analysis, we were to formulate the discretization equations for u and p' and obtain the values of uA, uB, and P2 using the initial guess of F or ρuA to be 5. Iterative solution methods were to be used and, in this case, the SIMPLE algorithm was used. This algorithm uses the continuity equation to show mass residuals resulting from our obtained values in the desired scenario. When the convergence is not met, a pressure correction is used to correct the pressure of that step and resolve for the unknown parameters. In simple terms, this process is a guess and check method for the pressure on a staggered grid.

From our initial guesses we were able to obtain a mass residual of zero with 9 to 10 iterations. Though the convergence rate of the mass residuals and the convergence and stability of the pressure profile was high, we did not maintain stability in the velocity profiles as seen in Figure 5. These oscillations are not ideal when looking for an accurate solution of the velocity.

Relaxation parameters were implemented to under-relax the solution with the goal of better stabilizing and converging across all parameters. Under-relaxation was used meaning the relaxation factor for velocity and pressure was kept below 1. When the factors are both equal to one, the solution remains the same as the initial solution. Reducing only the velocity or pressure relaxation factor as well as both at the same time was explored. The best combination of relaxation was concluded to be $\alpha_u = 0.6$ and $\alpha_p = 0.38$. This was found to be true because of a few factors. First, the solution satisfies the continuity equation and does so at a rate of high convergence. Looking at the velocity at both points A and B, the velocity is stable and converges quickly yielding solutions of be $u_A = 2$ and $u_B = 6$. Lastly, the pressure at point 2 is stable and converges quickly as well yielding a solution of $p_2 = 24$. High rates of convergence allow for shorter computation times which equates to less money put into the problem. A stable solution increases the accuracy of the solution. Many times, a compromise must be weighed to determine if the desire is for a quick solution or an accurate solution. We were able to achieve a combination of both, though the convergence did take a bit longer with than the initial ten iterations but is circumvented by the accuracy and stability of the solution.

Analysis

Solving any fluid dynamics problem yields the use of the fundamental equations of the Conservation of Mass, Momentum, and Energy. The conservation of mass is also referred to as the continuity equation which says that for constant density fluids within a control volume, the mass is conserved (what comes into the control volume must leave). The equation can be derived from the Eulerian approach

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) = 0 \quad (1.2)$$

When considering a Cartesian coordinate system of (x, y, z), we can represent the velocity vector components as (u, v, w) respectively. Equation (1.2) becomes

$$\frac{\partial \rho}{\partial x} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \quad (1.3)$$

Also, from our assumptions, we are considering a one-dimensional flow in the x directions with a nozzle that changes cross-sectional areas, which yields our final continuity equation that is given from equation (1.0) and first Governing Equation of

$$\frac{d}{dx}(\rho A u) = 0 \quad (1.4)$$

We can also consider $\rho A u = F$, so we can reduce Equation 1.4 to

$$\frac{d}{dx}F = 0 \quad (1.5)$$

The Conservation of momentum is directly related to Newtons Second Law which is states that the amount of momentum within the control volume remains constant. Application of this physical law gives the momentum equation that is represented by a vector equation of the mass multiplied by the velocity. We will consider the momentum in only the x direction as we are in a one-dimensional domain.

X – Momentum $$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2}\right) + \rho g_x \quad (1.6)$$

The left side of the equations represent the inertia. On the right-hand side of the equation, we have a negative pressure gradient, $\frac{\partial p}{\partial x}$, which states that we need the pressure to decrease for the fluid to accelerate, the viscosity, μ, the wall behavior from the shear stress at the walls, $\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x^2}\right)$, and the body force, ρg_x.

From the momentum equations, we can derive the Governing Equation for motion as shown below. In this case, we will only consider the x-momentum due to the one-dimensionality of problem. Also, from our assumptions we have an incompressible flow, no external forces, the flow is steady state, but we have a variable volume and we obtain the equation below as in the given

$$\frac{d}{dx}(\rho A u)u = -A\frac{dp}{dx} \quad (1.7)$$

With consideration of $\rho A u = F$, we can rewrite equation (1.7) to be

$$F\frac{du}{dx} = -A\frac{dp}{dx} \tag{1.8}$$

As discussed in the Introduction/Assumptions, we will be using an iterative method called the SIMPLE algorithm. The outline for this process is simply described in the figure below

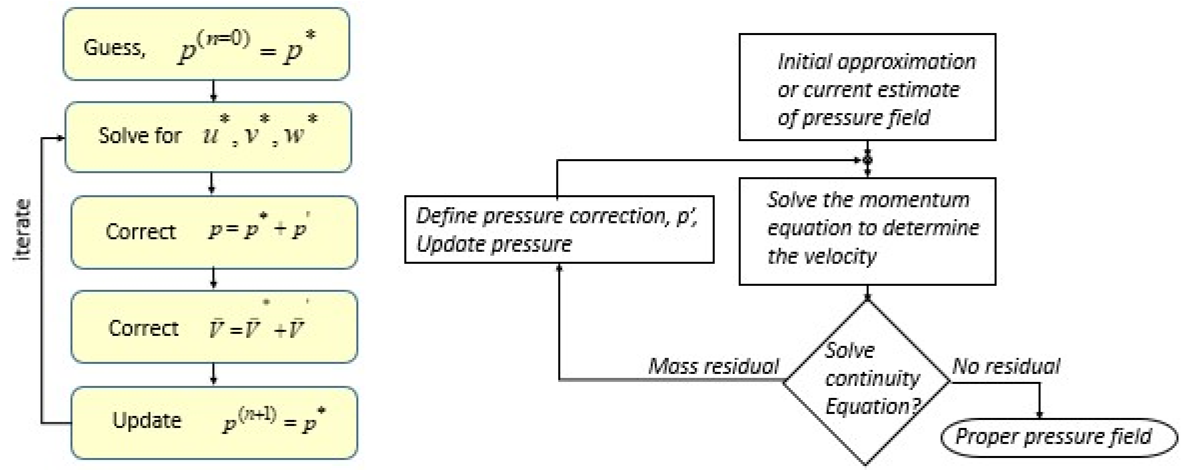

Figure 2: SIMPLE Method Outline

SIMPLE is an acronym for Semi-Implicit Pressure Linked Equation. The process starts with an initial guess for the pressure. This will lead to estimates of the velocity in each direction using the momentum equation (in our case just the u direction). These will yield new flow rates, F, can and then we evaluate the continuity equation for convergence or a mass residual of zero. If we end up with a mass residual, then continuity is not satisfied and we must make a pressure correction, which will lead to corrected velocities. With these parameter updates, we will update the original pressure to the corrected pressure and solve for the velocities again. This iterative process continues until the continuity is satisfied.

In our case, we are given an initial guess of F and we can then solve for the initial approximations at the internal node of velocities u_A and u_B and the pressure of point two, p_2.

$$F = 5 = \rho A u \tag{1.9}$$

We can calculate our initial calculations using Equation 1.9 yielding

$$u_A = \frac{5}{3}, \qquad u_B = 5, \qquad p_2 = 25$$

Back to consideration of the momentum equations, generally we can solve the momentum equation by using a general variable and adoption of the staggered grid. An example of a staggered control volume is shown below.

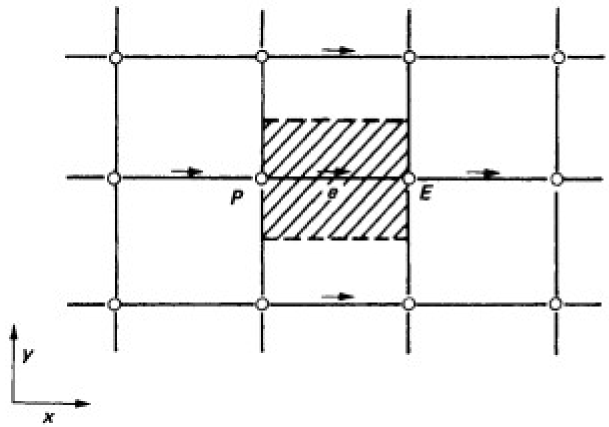

Figure 3: Staggered Grid Example

In this way the control volume is staggered when compared to the control volume that is seen around the primary point P. Using the staggered grid, we are able to calculate the acting pressure force for velocity, u, on the control volume using the difference between the two nodes of P and E. The discretization equation for Figure 3 is as seen below.

$$a_e u_e = \sum a_{nb} u_{nb} + b + A_e(p_P - p_E) \tag{1.10}$$

Since the nozzle problem at hand is dealing with a one-dimensional, we will have two u terms of interest. The ending term in Equation 1.10 represents the pressure for acting on the control volume. Having coefficient, A, is important as our nozzle has a variable area on which the pressure difference is acting upon, which are given at section A and B in Figure 1.

Going through iterations allows us to calculate the correct pressure field to subsequently find the velocity field that will satisfy the continuity equation. With the pressure corrections and guessed pressures, it is inevitable to have what are called imperfect velocities. The imperfect coefficients are typically represented by an asterix such as p^* and u^* for imperfect pressure and velocities. This can be directly brought into the discretization equation 1.10 as shown below

$$a_e {u^*}_e = \sum a_{nb} {u^*}_{nb} + b + A_e({p^*}_P - {p^*}_E) \tag{1.11}$$

When the velocities do not satisfy the continuity equation, we must correct the pressure using the equation below

$$p = p^* + p' \tag{1.12}$$

The p* in this equation is the imperfect pressure and p' is the pressure correction. This correction in pressure will lead to effected velocities. The velocities will also have a correction similar to the pressure given by

$$u = u^* + u' \tag{1.13}$$

To obtain the corrected discretization equation, we can subtract Equation 1.11 from Equation 1.10 which results in

$$a_e {u'}_e = \sum a_{nb} {u'}_{nb} + A_e({p'}_P - {p'}_E) \tag{1.14}$$

From Equation 1.14, we can obtain the final correction equation of

$$u'_e = \frac{A_e}{a_e}(p'_P - p'_E) \tag{1.15}$$

Or in terms of the imperfect pressure knowing Equation (1.13)

$$u_e = u^*_e \frac{A_e}{a_e}(p'_P - p'_E) \tag{1.16}$$

Notice the summation term was dropped. This is due to computational simplicity that will not affect the end result. If this term were kept, then the equations would become very complex when trying to characterize the pressure within the velocity correction equations. As a first approximation, the neighboring components of the velocity correction are usually unknown and so they are typically neglected. Also, due to the very nature of the SIMPLE algorithm, the converged solution contains no error when removing the summation term. Since this algorithm is described as semi-implicit, this term is not needed. This summation term has an implicit effect on the pressure correction on velocity which again is not considered.

In terms of the problem at hand and the imperfect velocities at sections A and B

$$u_A^* = \frac{A_A(p_1 - p_2^*)}{F} \tag{1.17}$$

$$u_B^* = \frac{A_B(p_1 - p_2^*) + F u_A^*}{F} \tag{1.18}$$

The pressure correction equation is found by setting up continuity at internal nodes as shown below

$$A_A u_A = A_B u_B \tag{1.18}$$

From Equation 1.13

$$A_A(u_A^* + u_A') = A_B(u_B^* + u_B') \tag{1.19}$$

From Equation 1.15

$$A_A\left(u_A^* + \frac{A_A}{F}(p'_1 - p'_2)\right) = A_B\left(u_B^* + \frac{A_B}{F}(p'_2 - p'_3)\right) \tag{1.20}$$

Rearranging

$$p_2' = \frac{F(A_A u_A^* - A_B u_B^*)}{A_A^2 + A_B^2} \tag{1.21}$$

The flow rates can be calculated using the equations below

$$F_A = \frac{A_A^2(p_1 - p_2^*)}{F} \tag{1.22}$$

$$F_B = A_B u_A^* + \frac{A_B^2(p_2^* - p_3)}{F} \tag{1.23}$$

Using the continuity equation to check for convergence or a mass residual of zero, we can use

$$\{R\}_{n\to\infty} = |F_A - F_B| \to 0 \tag{1.24}$$

Using all of the defined equations, the SIMPLE Algorithm will be used referencing Figure 2 as follows

1. Use the guessed value of F and corresponding velocities and pressure (values found from Equation 1.9)
2. Calculate new velocities using Equations 1.17 and 1.18
3. Calculate the pressure correction using Equation 1.21
4. Calculate the new pressure using Equation 1.12
5. Update the flow rates using Equations 1.22 and 1.23
6. Check convergence using equation 1.24.
7. If step six does not yield a mass residual of zero, then use the newly acquired flow rate at section A and calculate the new velocities starting at step 2.
8. Repeat until mass residual goes to zero

Given Conditions				
ρ	p1	p3	A_A	A_B
1	28	0	3	1

Iteration	F	F_A	F_B	p2	p'2	u_A	u_B	Residual
0	5	5	5	25	-	1.666667	5	-
1	5.000	5.400	6.800	24.300	-0.700	1.8	6.8	1.40000
2	5.400	6.660	7.080	24.090	-0.210	2.22	7.08	0.42000
3	6.660	6.517	6.633	24.027	-0.063	2.172222	6.633333	0.11667
4	6.517	5.369	5.397	24.008	-0.019	1.78964	5.397297	0.02838
5	5.369	5.513	5.522	24.002	-0.006	1.837703	5.521811	0.00870
6	5.513	6.701	6.704	24.001	-0.002	2.233729	6.704355	0.00317
7	6.701	6.529	6.530	24.000	-0.001	2.176233	6.529625	0.00093
8	6.529	5.372	5.372	24.000	0.000	1.79063	5.372117	0.00023
9	5.372	5.514	5.514	24.000	0.000	1.838008	5.514095	0.00007
10	5.514	6.702	6.702	24.000	0.000	2.23384	6.701546	0.00003

Table 1: Initial Solution SIMPLE Algorithm (No Relaxation)

We can see that it took about ten iterations for convergence to occur (still not fully), seen by the residual approaching zero. This can also be viewed graphically below.

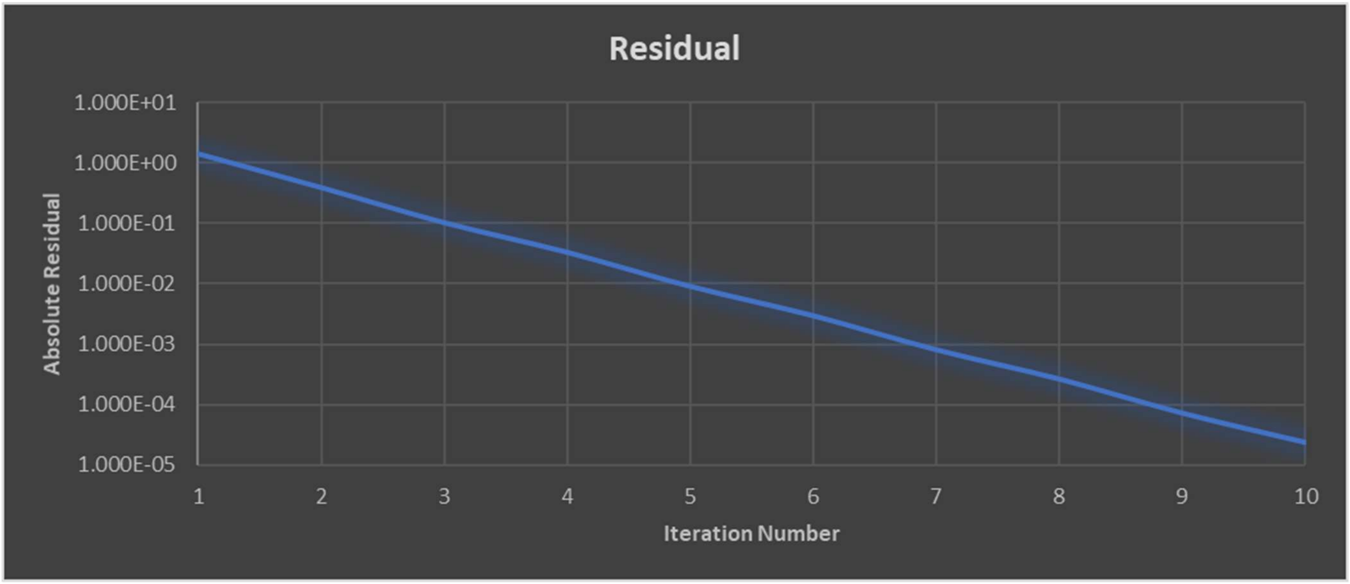

Figure 4: Residual Plot of Initial SIMPLE Algorithm (No Relaxation)

Though we have taken a short time to near convergence, the velocities have not reached their true value. We are fluctuating around the true values as seen below.

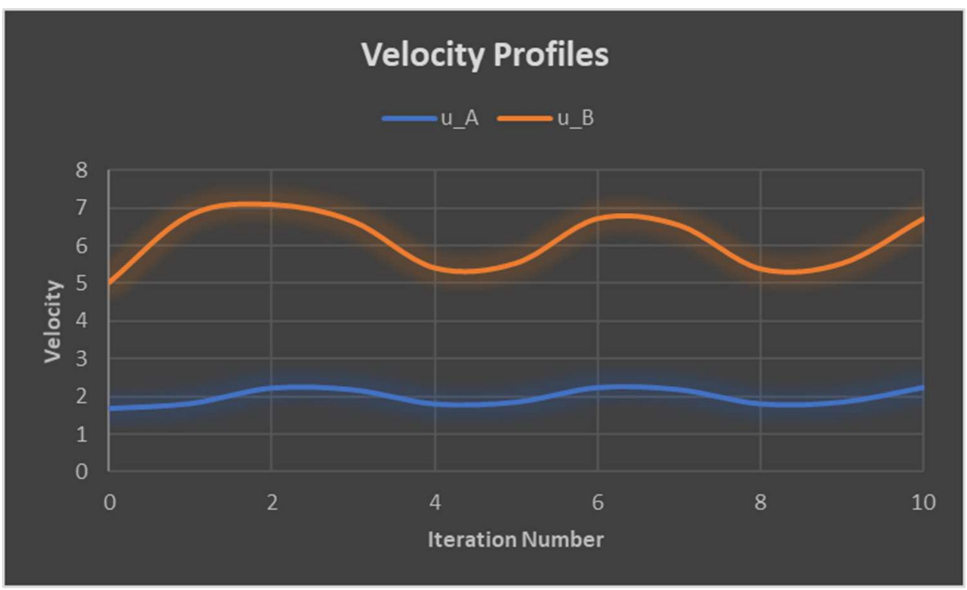

Figure 5: Velocity Profile, Initial SIMPLE Algorithm (No Relaxation)

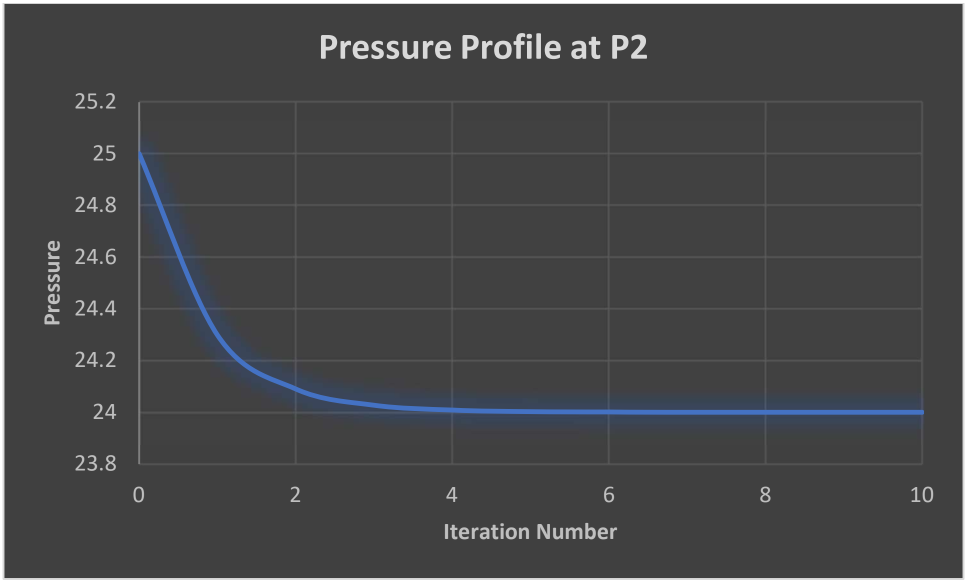

Figure 6: Pressure Profile at P2, Initial SIMPLE Algorithm (No Relaxation)

With the oscillations around the target solutions, we must employ relaxation factors to the velocity to find a balance of a quick convergence and an accurate true target solution. The pressure can be seen to converge to a stable solution. With an iterative process like this, we must find a solution that both converges and is also stable on all parameters of interest.

With our initial solution, we can see that our solution is converging, but the solution is unstable on velocity as seen in Figure 5. As from above, this situation calls for the use of an under-relaxation factor. In applying this to our solution, we can see an example of

$$u_{n+1} = \alpha u_{n+1} + (1 - \alpha) u_n \tag{1.25}$$

Where α is the relaxation factor. When we have an $\alpha < 1$, and apply it to equation 1.25, the final value on the left-hand side will approach the target solution more slowly. The residuals of ($u_{n+1} - u_n$) will be seen to decrease proportionally when compared to alpha. This results in a slower convergence, but a more stable solution that should not oscillate or overshoot the target solution.

In this problem set, we are having an issue with the stabilization of our target solution, so we would like to tune our solution using the relaxation factor while taking into consideration the rate of convergence. Relaxation factors do not always have a very clear definition on what to choose or how to relax your solution. Relaxation can take on the form of a guess and check instead of an exact solution. Below we will explore some under-relaxation factors to find the best balance between stability and rate of convergence.

Using relaxation parameters, α_p and α_u, representing the pressure relaxation and velocity relaxation respectively, we can update equations used in the SIMPLE Algorithm as shown below

$$F^n = \rho A u_{relax}^{n-1} \tag{1.26}$$

Updated velocity

$$u_A^n = \frac{A_A}{F^n}(p_1 - p_2^{n-1}{}_{relax}) \tag{1.27}$$

$$u_B^n = \frac{A_B}{F^n}\left(p_2^{n-1}{}_{relax} - p_3\right) + u_A^n \tag{1.28}$$

$$u_A^n{}_{relax} = \alpha_u u_A^n + (1-\alpha_u)u_A^{n-1}{}_{relax} \tag{1.29}$$

$$u_B^n{}_{relax} = \alpha_u u_B^n + (1-\alpha_u)u_B^{n-1}{}_{relax} \tag{1.30}$$

Update pressure correction

$$p_2'{}^n = \frac{F^n(A_A u_A^n{}_{relax} - A_B u_B^n{}_{relax})}{A_A^2 + A_B^2} \tag{1.31}$$

Updated pressure with pressure correction

$$p_2^n{}_{relax} = p_2^{n-1}{}_{relax} + \alpha_p p_2'{}^2 \tag{1.32}$$

Updated flow rates

$$F_A = \alpha_u F_A^{new} + (1-\alpha_u)F_A^{old} \tag{1.33}$$

$$F_B = \alpha_u F_B^{new} + (1-\alpha_u)F_B^{old} \tag{1.34}$$

$$F_A^{new} = \frac{A_A^2(p_1 - p_2^*)}{F} \tag{1.35}$$

$$F_A^{old} = \frac{A_A^2(p_1 - p_2^{old})}{F} \tag{1.36}$$

$$F_B^{new} = A_B u_A^* + \frac{A_B^2(p_2^* - p_3)}{F} \tag{1.37}$$

$$F_B^{old} = A_B u_A^* + \frac{A_B^2(p_2^{old} - p_3)}{F} \tag{1.38}$$

Exploration of under-relaxation factors can be seen below

Relaxation Parameters				
au	1		ap	1

Iteration	F	F_A	F_A_R	F_B	F_B_R	p2_R	p'2	u_A	u_A_R	u_B	u_B_R	Residual
0	5	5	5	5	5	25	-	1.6667	1.6667	5.0000	5.0000	-
1	5.000	5.400	5.400	6.800	6.800	24.300	-0.700	1.8000	1.8000	6.8000	6.8000	1.4000000
2	5.400	6.167	6.167	6.556	6.556	24.090	-0.210	2.0556	2.0556	6.5556	6.5556	0.3888889
3	6.167	5.706	5.706	5.809	5.809	24.027	-0.063	1.9022	1.9022	5.8086	5.8086	0.1021622
4	5.706	6.266	6.266	6.299	6.299	24.008	-0.019	2.0887	2.0887	6.2991	6.2991	0.0331202
5	6.266	5.734	5.734	5.743	5.743	24.002	-0.006	1.9112	1.9112	5.7427	5.7427	0.0090488
6	5.734	6.275	6.275	6.278	6.278	24.001	-0.002	2.0916	2.0916	6.2779	6.2779	0.0029667
7	6.275	5.736	5.736	5.737	5.737	24.000	-0.001	1.9120	1.9120	5.7369	5.7369	0.0008132
8	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2760	6.2760	0.0002669
9	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7364	5.7364	0.0000732
10	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000240
11	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000066
12	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000022
13	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000006
14	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000002
15	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000001
16	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000000
17	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000000
18	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000000
19	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000000
20	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000000
21	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000000
22	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000000
23	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000000
24	5.736	6.276	6.276	6.276	6.276	24.000	0.000	2.0919	2.0919	6.2758	6.2758	0.0000000
25	6.276	5.736	5.736	5.736	5.736	24.000	0.000	1.9121	1.9121	5.7363	5.7363	0.0000000

Table 2: Relaxation Search, au=1 ap=1

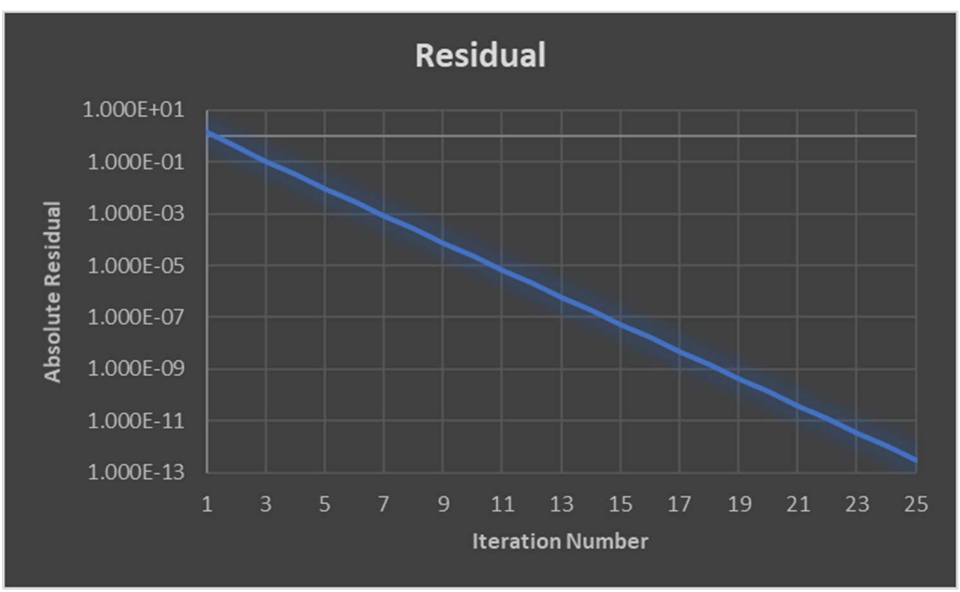

Figure 7: Residual Graph, Relaxation Search, au=1 ap=1

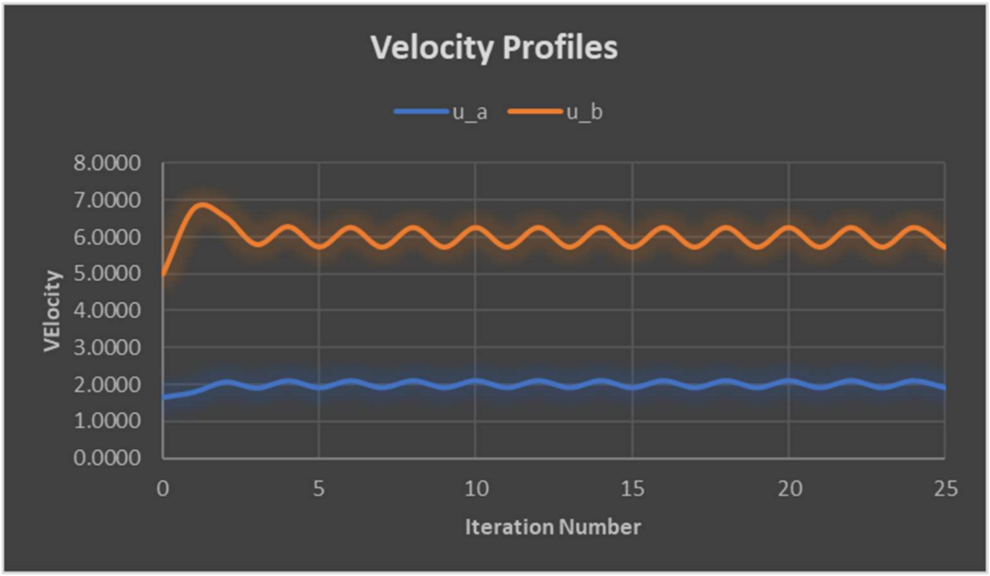

Figure 8: Velocity Profile, Relaxation Search, au=1 ap=1

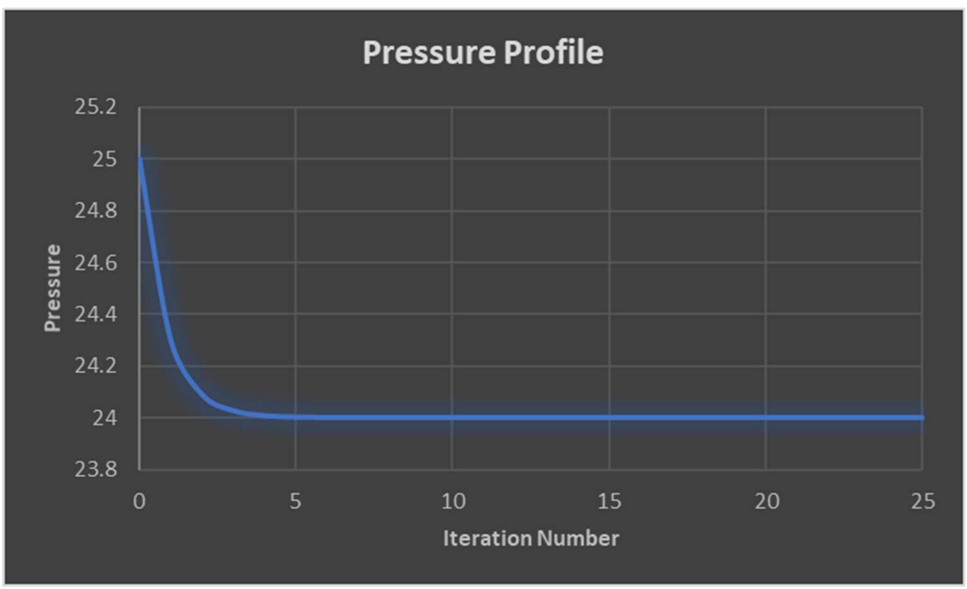

Figure 9: Pressure Profile at P2, Relaxation Search, au=1 ap=1

The start of the relaxation search uses a relaxation of one, for both the velocity and pressure relaxation, which yields the same results as the initial solution, but as we can see there are more iterations to study the velocity profiles more. From figures above, we can see that the convergence is very quick, the pressure profile is stable, but the velocity profiles are not stable as they are oscillating around the target solution. The velocity at B is fluctuating around a velocity of 6 and velocity at A is fluctuating around a velocity of 2. Since we are not stable, we should explore other options of the relaxation factors.

Relaxation Parameters				
au	0.5		ap	1

Iteration	F	F_A	F_A_R	F_B	F_B_R	p2_R	p'2	u_A	u_A_R	u_B	u_B_R	Residual
0	5	5	5	5	5	25	-	1.6667	1.6667	5.0000	5.0000	-
1	5.000	5.400	5.200	6.800	5.900	24.650	-0.350	1.8000	1.7333	6.8000	5.9000	0.7000000
2	5.200	5.798	5.499	6.673	6.287	24.241	-0.409	1.9327	1.8330	6.6731	6.2865	0.7875000
3	5.499	6.153	5.826	6.459	6.373	23.940	-0.301	2.0510	1.9420	6.4591	6.3728	0.5468222
4	5.826	6.272	6.049	6.200	6.286	23.802	-0.138	2.0907	2.0164	6.1999	6.2863	0.2372459
5	6.049	6.247	6.148	6.017	6.152	23.799	-0.002	2.0822	2.0493	6.0169	6.1516	0.0038177
6	6.148	6.150	6.149	5.921	6.036	23.868	0.069	2.0499	2.0496	5.9210	6.0363	0.1123678
7	6.149	6.048	6.098	5.898	5.967	23.949	0.081	2.0159	2.0327	5.8977	5.9670	0.1311209
8	6.098	5.979	6.038	5.920	5.944	24.007	0.058	1.9929	2.0128	5.9202	5.9436	0.0948459
9	6.038	5.952	5.995	5.960	5.952	24.033	0.026	1.9839	1.9983	5.9595	5.9516	0.0434737
10	5.995	5.955	5.975	5.994	5.973	24.035	0.001	1.9851	1.9917	5.9939	5.9727	0.0024331
11	5.975	5.973	5.974	6.013	5.993	24.023	-0.011	1.9910	1.9914	6.0134	5.9931	0.0190058
12	5.974	5.991	5.983	6.018	6.006	24.009	-0.014	1.9971	1.9942	6.0183	6.0057	0.0230758
13	5.983	6.003	5.993	6.014	6.010	23.999	-0.010	2.0011	1.9977	6.0143	6.0100	0.0170264
14	5.993	6.008	6.001	6.007	6.009	23.994	-0.005	2.0027	2.0002	6.0073	6.0087	0.0080433
15	6.001	6.008	6.004	6.001	6.005	23.994	0.000	2.0026	2.0014	6.0013	6.0050	0.0007407
16	6.004	6.005	6.005	5.998	6.001	23.996	0.002	2.0016	2.0015	5.9978	6.0014	0.0031676
17	6.005	6.002	6.003	5.997	5.999	23.998	0.002	2.0006	2.0010	5.9968	5.9991	0.0040130
18	6.003	6.000	6.001	5.997	5.998	24.000	0.002	1.9998	2.0004	5.9975	5.9983	0.0030314
19	6.001	5.999	6.000	5.999	5.998	24.001	0.001	1.9995	2.0000	5.9987	5.9985	0.0014796
20	6.000	5.999	5.999	6.000	5.999	24.001	0.000	1.9995	1.9998	5.9997	5.9991	0.0001857
21	5.999	5.999	5.999	6.000	6.000	24.001	0.000	1.9997	1.9997	6.0004	5.9997	0.0005263
22	5.999	6.000	5.999	6.001	6.000	24.000	0.000	1.9999	1.9998	6.0005	6.0001	0.0006981
23	5.999	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	1.9999	6.0005	6.0003	0.0005397
24	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0001	2.0000	6.0002	6.0003	0.0002716
25	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0001	2.0000	6.0001	6.0002	0.0000425

Table 3: Relaxation Search, au=0.5 ap=1

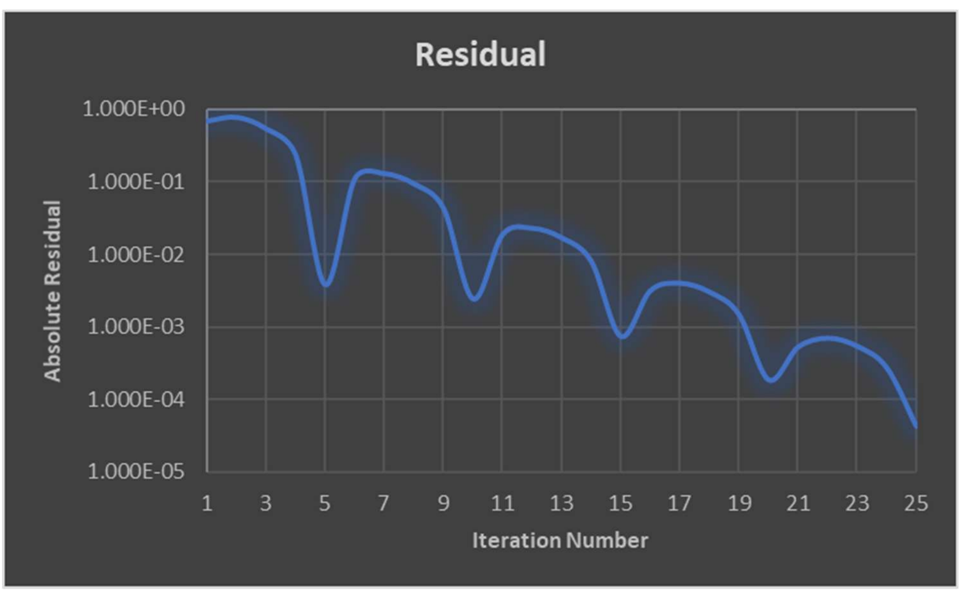

Figure 10: Residual Graph, Relaxation Search, au=0.5 ap=1

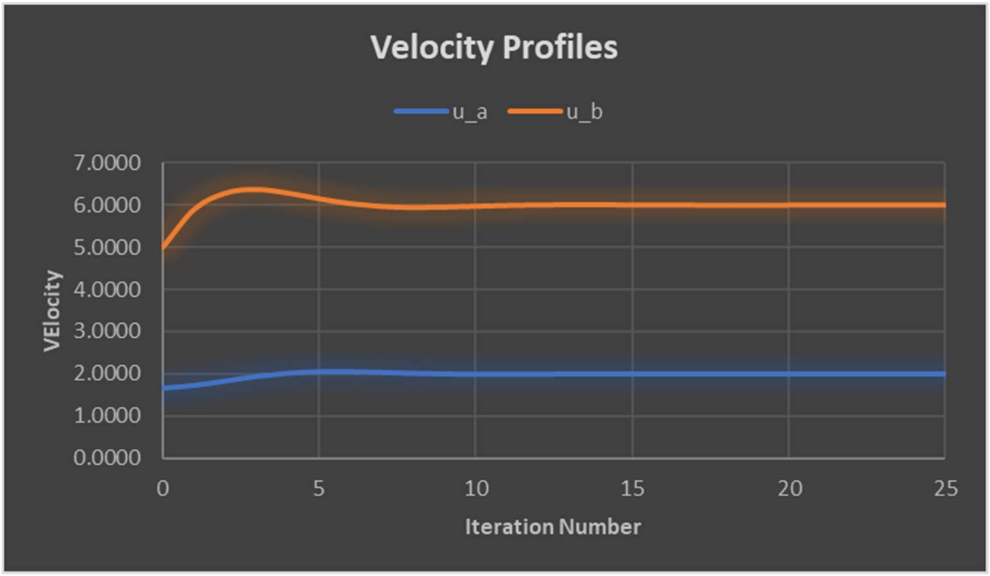

Figure 11: Velocity Profiles, Relaxation Search, au=0.5 ap=1

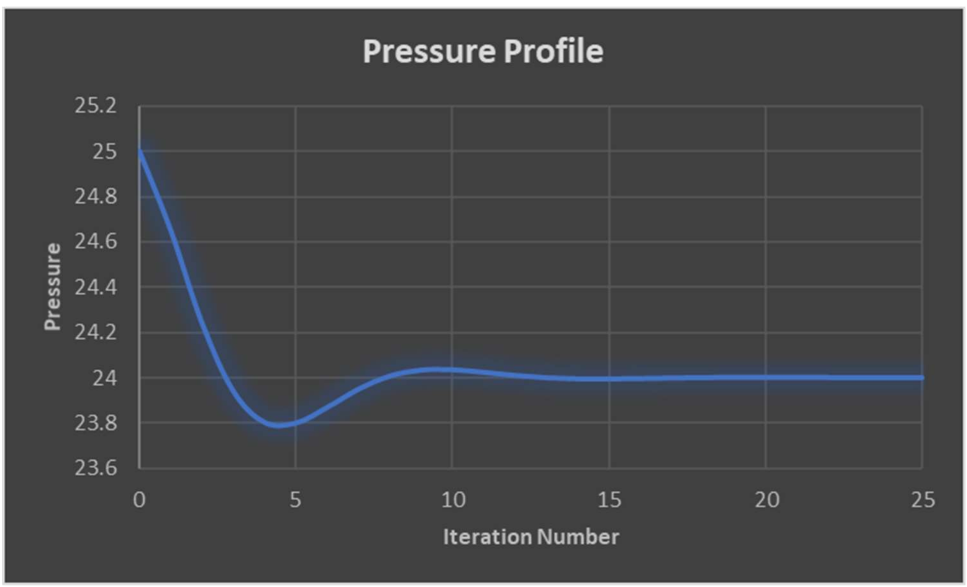

Figure 12: Pressure Profile, Relaxation Search, au=0.5 ap=1

Maintaining a pressure relaxation of 1 and cutting the velocity relaxation in half (0.5) can be seen to create stabilization in the velocity profiles (Figure 11). It does have a negative impact on the pressure profile showing an undershoot and overshoot before stabilizing (Figure 12), but this is could be a compromise if convergence occurs quickly. As we can see from the residual graph (Figure 10), the residuals fluctuate quite a bit and take too long to reach a zero-mass residual.

Relaxation Parameters				
au	0.5		ap	0.5

Iteration	F	F_A	F_A_R	F_B	F_B_R	p2_R	p'2	u_A	u_A_R	u_B	u_B_R	Residual
0	5	5	5	5	5	25	-	1.6667	1.6667	5.0000	5.0000	-
1	5.000	5.400	5.200	6.800	5.900	24.825	-0.350	1.8000	1.7333	6.8000	5.9000	0.7000000
2	5.200	5.495	5.348	6.606	6.253	24.590	-0.471	1.8317	1.7825	6.6058	6.2529	0.9052885
3	5.348	5.740	5.544	6.511	6.382	24.365	-0.448	1.9132	1.8479	6.5115	6.3822	0.8385536
4	5.544	5.901	5.722	6.362	6.372	24.185	-0.360	1.9669	1.9074	6.3621	6.3721	0.6499821
5	5.722	6.000	5.861	6.227	6.299	24.060	-0.251	2.0000	1.9537	6.2266	6.2994	0.4383003
6	5.861	6.050	5.956	6.122	6.211	23.985	-0.149	2.0168	1.9852	6.1218	6.2106	0.2548893
7	5.956	6.067	6.011	6.050	6.130	23.950	-0.071	2.0224	2.0038	6.0496	6.1301	0.1187191
8	6.011	6.064	6.038	6.005	6.068	23.941	-0.018	2.0213	2.0125	6.0053	6.0677	0.0301314
9	6.038	6.051	6.044	5.982	6.025	23.947	0.012	2.0170	2.0148	5.9823	6.0250	0.0192857
10	6.044	6.036	6.040	5.974	5.999	23.959	0.025	2.0119	2.0133	5.9737	5.9994	0.0405849
11	6.040	6.022	6.031	5.974	5.987	23.972	0.027	2.0072	2.0103	5.9740	5.9867	0.0441493
12	6.031	6.011	6.021	5.979	5.983	23.984	0.023	2.0036	2.0069	5.9786	5.9826	0.0382298
13	6.021	6.004	6.012	5.985	5.984	23.992	0.017	2.0012	2.0041	5.9846	5.9836	0.0285955
14	6.012	5.999	6.006	5.990	5.987	23.998	0.011	1.9998	2.0019	5.9904	5.9870	0.0187806
15	6.006	5.997	6.002	5.995	5.991	24.001	0.006	1.9991	2.0005	5.9949	5.9910	0.0105878
16	6.002	5.997	5.999	5.998	5.995	24.003	0.003	1.9989	1.9997	5.9981	5.9945	0.0046381
17	5.999	5.997	5.998	6.000	5.997	24.003	0.001	1.9990	1.9994	6.0000	5.9973	0.0008510
18	5.998	5.998	5.998	6.001	5.999	24.002	-0.001	1.9992	1.9993	6.0010	5.9991	0.0011918
19	5.998	5.998	5.998	6.001	6.000	24.002	-0.001	1.9995	1.9994	6.0013	6.0002	0.0020046
20	5.998	5.999	5.999	6.001	6.001	24.001	-0.001	1.9997	1.9995	6.0012	6.0007	0.0020602
21	5.999	6.000	5.999	6.001	6.001	24.001	-0.001	1.9999	1.9997	6.0010	6.0008	0.0017274
22	5.999	6.000	5.999	6.001	6.001	24.000	-0.001	2.0000	1.9998	6.0007	6.0008	0.0012587
23	5.999	6.000	6.000	6.000	6.001	24.000	0.000	2.0000	1.9999	6.0004	6.0006	0.0008040
24	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	2.0000	6.0002	6.0004	0.0004360
25	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0001	2.0000	6.0001	6.0002	0.0001757

Table 4: Relaxation Search, au=0.5 ap=0.5

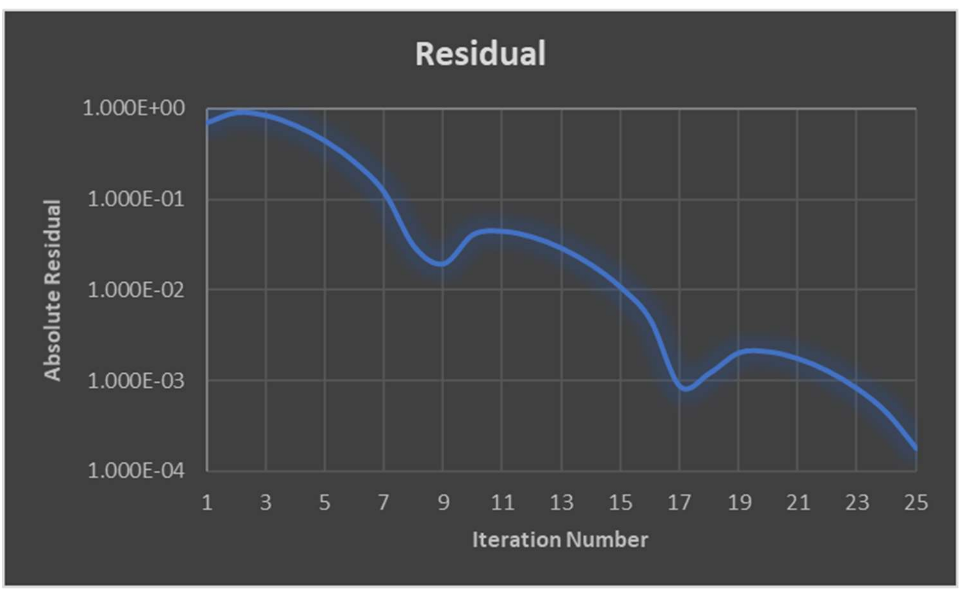

Figure 14: Residual Graph, Relaxation Search, au=0.5 ap=0.5

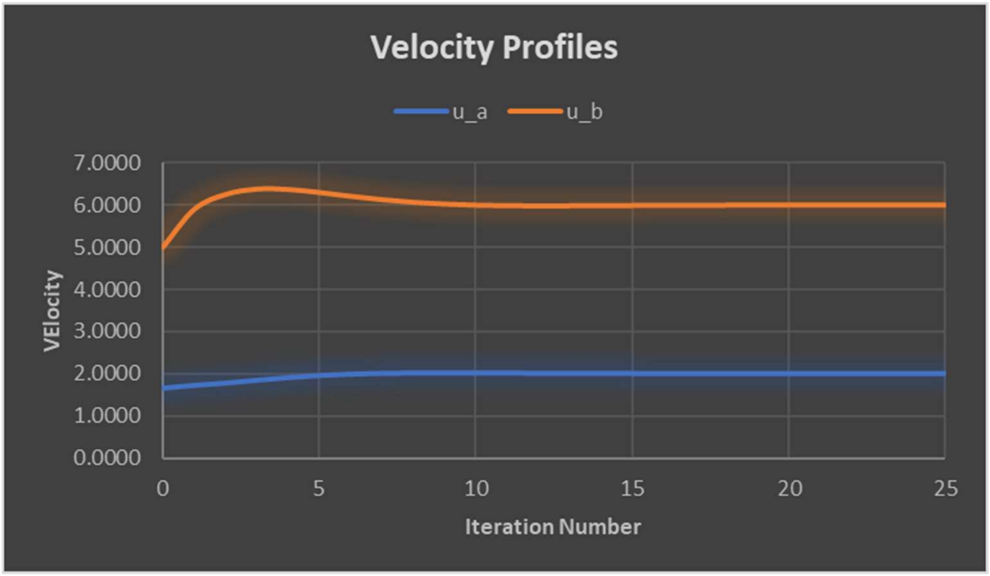

Figure 14: Velocity Profiles, Relaxation Search, au=0.5 ap=0.5

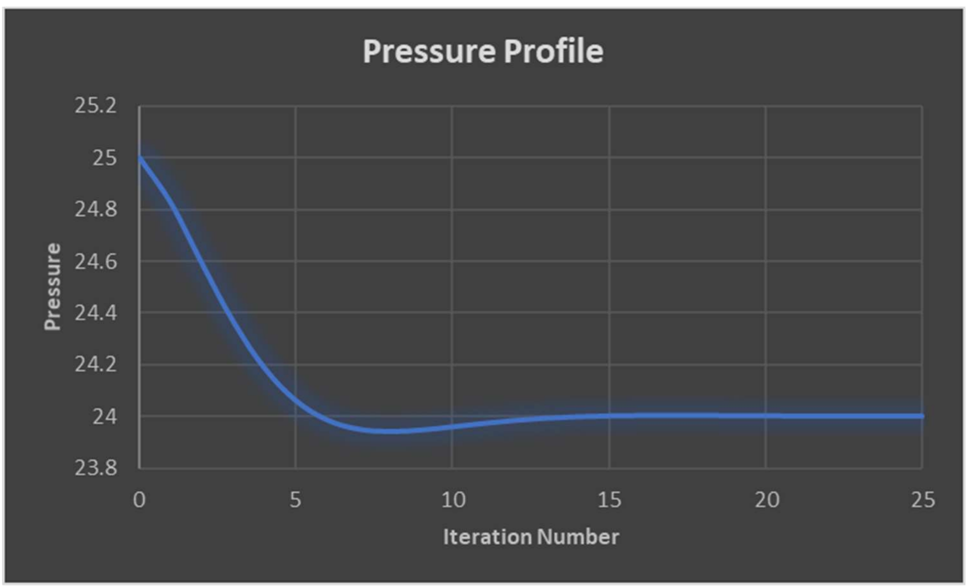

Figure 15: Pressure Profile, Relaxation Search, au=0.5 ap=0.5

Keeping the velocity relaxation at 0.5 and cutting the pressure relaxation in half (0.5), we get a very good solution, but it would be ideal if the rate of convergence was quicker. There are about 3 fluctuations before getting to a mass residual of zero (Figure 13). The velocity has maintained its stabilization (Figure 14) and the pressure has better stabilized, though there is still an undershoot of the pressure before stabilization (Figure 15).

Relaxation Parameters				
au	0.1		ap	1

Iteration	F	F_A	F_A_R	F_B	F_B_R	p2_R	p'2	u_A	u_A_R	u_B	u_B_R	Residual
0	5	5	5	5	5	25	-	1.6667	1.6667	5.0000	5.0000	-
1	5.000	5.400	5.040	6.800	5.180	24.930	-0.070	1.8000	1.6800	6.8000	5.1800	0.1400000
2	5.040	5.482	5.084	6.774	5.339	24.801	-0.129	1.8274	1.6947	6.7738	5.3394	0.2551667
3	5.084	5.662	5.142	6.765	5.482	24.629	-0.173	1.8874	1.7140	6.7655	5.4820	0.3399870
4	5.142	5.901	5.218	6.757	5.609	24.427	-0.201	1.9670	1.7393	6.7567	5.6095	0.3915537
5	5.218	6.162	5.312	6.736	5.722	24.213	-0.214	2.0542	1.7708	6.7356	5.7221	0.4097090
6	5.312	6.415	5.423	6.696	5.819	24.003	-0.211	2.1384	1.8075	6.6963	5.8195	0.3968600
7	5.423	6.635	5.544	6.638	5.901	23.809	-0.194	2.2115	1.8479	6.6379	5.9013	0.3575088
8	5.544	6.804	5.670	6.563	5.967	23.644	-0.165	2.2681	1.8900	6.5627	5.9675	0.2976068
9	5.670	6.915	5.794	6.475	6.018	23.517	-0.127	2.3050	1.9315	6.4750	6.0182	0.2238625
10	5.794	6.963	5.911	6.380	6.054	23.434	-0.083	2.3211	1.9704	6.3797	6.0544	0.1431041
11	5.911	6.952	6.015	6.282	6.077	23.397	-0.037	2.3173	2.0051	6.2816	6.0771	0.0617567
12	6.015	6.886	6.102	6.185	6.088	23.406	0.009	2.2954	2.0341	6.1851	6.0879	0.0145443
13	6.102	6.775	6.170	6.094	6.088	23.456	0.050	2.2584	2.0566	6.0939	6.0885	0.0812108
14	6.170	6.629	6.216	6.011	6.081	23.539	0.083	2.2097	2.0719	6.0114	6.0808	0.1348452
15	6.216	6.460	6.240	5.940	6.067	23.647	0.108	2.1532	2.0800	5.9402	6.0667	0.1732904
16	6.240	6.279	6.244	5.882	6.048	23.769	0.122	2.0930	2.0813	5.8825	6.0483	0.1956052
17	6.244	6.099	6.229	5.840	6.027	23.895	0.126	2.0330	2.0765	5.8397	6.0274	0.2019800
18	6.229	5.931	6.200	5.813	6.006	24.015	0.121	1.9770	2.0665	5.8128	6.0060	0.1936061
19	6.200	5.785	6.158	5.802	5.986	24.122	0.107	1.9282	2.0527	5.8019	5.9856	0.1725089
20	6.158	5.667	6.109	5.806	5.968	24.209	0.087	1.8891	2.0363	5.8062	5.9676	0.1413527
21	6.109	5.584	6.057	5.824	5.953	24.272	0.063	1.8615	2.0188	5.8244	5.9533	0.1032261
22	6.057	5.539	6.005	5.854	5.943	24.310	0.037	1.8464	2.0016	5.8540	5.9434	0.0614161
23	6.005	5.531	5.957	5.892	5.938	24.321	0.012	1.8437	1.9858	5.8921	5.9383	0.0191796
24	5.957	5.558	5.917	5.935	5.938	24.309	-0.012	1.8526	1.9725	5.9351	5.9379	0.0204735
25	5.917	5.614	5.887	5.979	5.942	24.276	-0.033	1.8713	1.9624	5.9793	5.9421	0.0549734

Table 5: Relaxation Search, au=0.1 ap=1

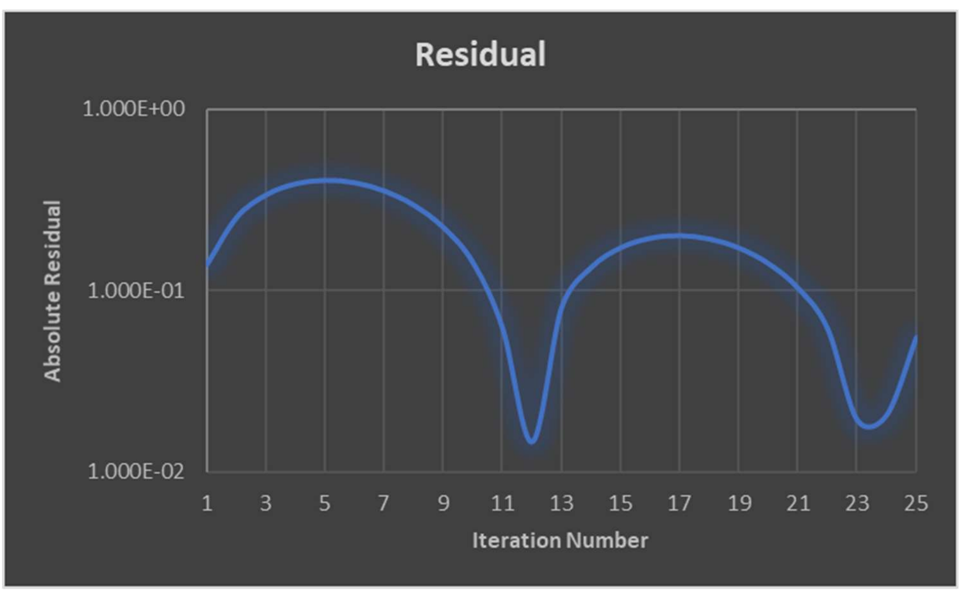

Figure 16: Residual Graph, Relaxation Search, au=0.1 ap=1

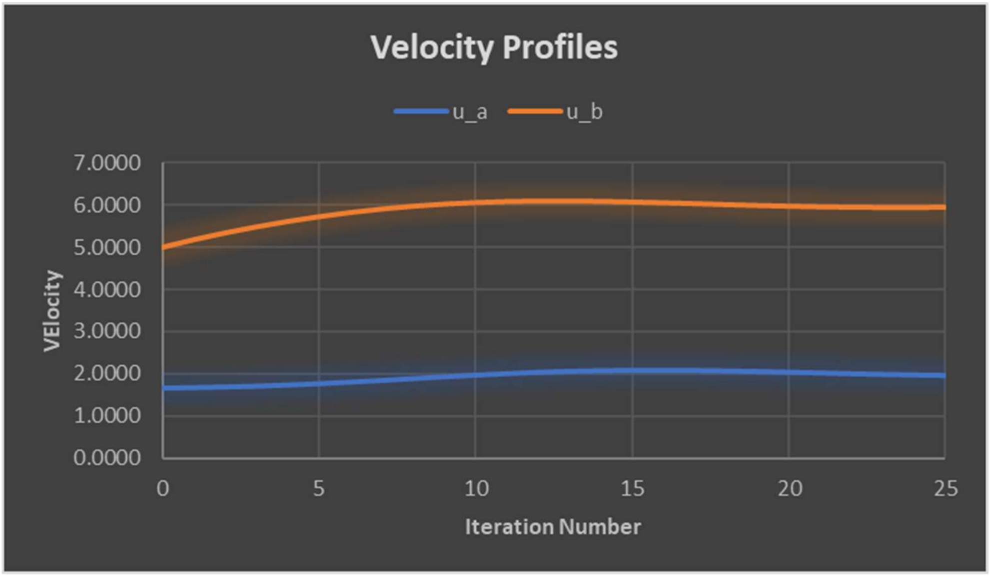

Figure 17: Velocity Profiles, Relaxation Search, au=0.1 ap=1

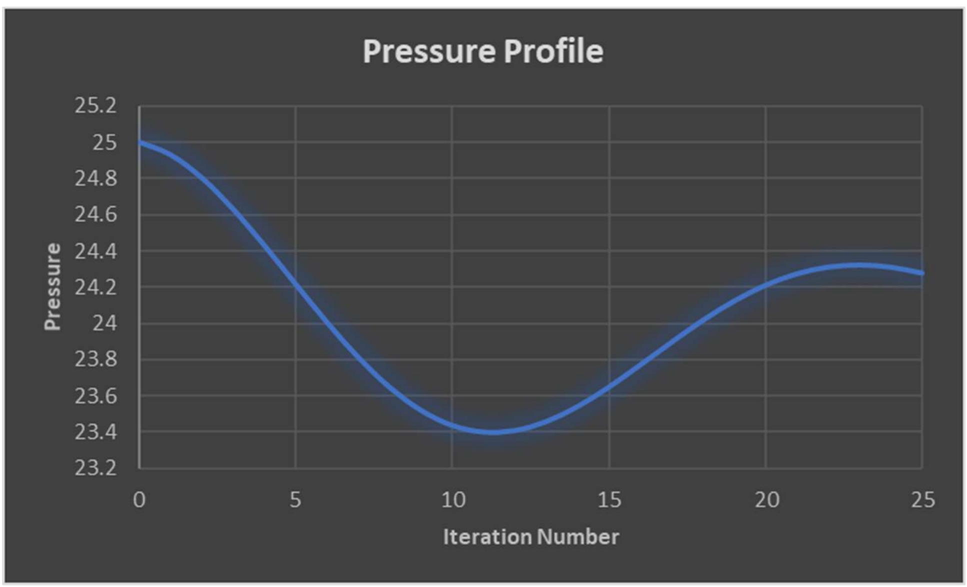

Figure 18: Pressure Profile, Relaxation Search, au=0.1 ap=1

Studying the cause of reducing the velocity relaxation to 0.1 and keeping the pressure relaxation at one shows a solution with a slow rate of convergence and an unstable pressure profile. The velocity profiles look to approach stability but are fluctuating around the target value in large periods and do not settle by the twenty-fifth iteration.

Relaxation Parameters				
au	0.1		ap	0.1

Iteration	F	F_A	F_A_R	F_B	F_B_R	p2_R	p'2	u_A	u_A_R	u_B	u_B_R	Residual
0	5	5	5	5	5	25	-	1.6667	1.6667	5.0000	5.0000	-
1	5.000	5.400	5.040	6.800	5.180	24.993	-0.070	1.8000	1.6800	6.8000	5.1800	0.1400000
2	5.040	5.370	5.073	6.749	5.337	24.980	-0.133	1.7899	1.6910	6.7488	5.3369	0.2639167
3	5.073	5.358	5.102	6.710	5.474	24.961	-0.189	1.7861	1.7005	6.7102	5.4742	0.3727101
4	5.102	5.362	5.128	6.680	5.595	24.937	-0.238	1.7872	1.7092	6.6801	5.5948	0.4672735
5	5.128	5.376	5.152	6.655	5.701	24.909	-0.281	1.7921	1.7175	6.6555	5.7009	0.5484572
6	5.152	5.400	5.177	6.634	5.794	24.877	-0.318	1.7998	1.7257	6.6342	5.7942	0.6170842
7	5.177	5.429	5.202	6.615	5.876	24.842	-0.349	1.8097	1.7341	6.6149	5.8763	0.6739601
8	5.202	5.463	5.228	6.596	5.948	24.805	-0.375	1.8210	1.7428	6.5962	5.9483	0.7198794
9	5.228	5.500	5.256	6.578	6.011	24.765	-0.395	1.8334	1.7519	6.5777	6.0112	0.7556275
10	5.256	5.540	5.284	6.559	6.066	24.724	-0.411	1.8465	1.7613	6.5587	6.0660	0.7819813
11	5.284	5.580	5.314	6.539	6.113	24.682	-0.423	1.8599	1.7712	6.5390	6.1133	0.7997077
12	5.314	5.620	5.344	6.518	6.154	24.639	-0.430	1.8734	1.7814	6.5185	6.1538	0.8095607
13	5.344	5.660	5.376	6.497	6.188	24.595	-0.434	1.8868	1.7919	6.4972	6.1881	0.8122785
14	5.376	5.700	5.408	6.475	6.217	24.552	-0.435	1.8999	1.8027	6.4751	6.2168	0.8085799
15	5.408	5.738	5.441	6.452	6.240	24.509	-0.432	1.9127	1.8137	6.4524	6.2404	0.7991605
16	5.441	5.775	5.475	6.429	6.259	24.466	-0.427	1.9249	1.8249	6.4292	6.2593	0.7846898
17	5.475	5.810	5.508	6.406	6.274	24.424	-0.419	1.9366	1.8360	6.4056	6.2739	0.7658082
18	5.508	5.843	5.542	6.382	6.285	24.383	-0.409	1.9476	1.8472	6.3819	6.2847	0.7431241
19	5.542	5.874	5.575	6.358	6.292	24.343	-0.397	1.9580	1.8583	6.3581	6.2920	0.7172123
20	5.575	5.903	5.608	6.334	6.296	24.305	-0.384	1.9677	1.8692	6.3344	6.2963	0.6886125
21	5.608	5.930	5.640	6.311	6.298	24.268	-0.369	1.9767	1.8800	6.3110	6.2977	0.6578282
22	5.640	5.955	5.671	6.288	6.297	24.233	-0.353	1.9851	1.8905	6.2880	6.2968	0.6253259
23	5.671	5.978	5.702	6.265	6.294	24.199	-0.335	1.9927	1.9007	6.2655	6.2936	0.5915360
24	5.702	5.999	5.732	6.244	6.289	24.168	-0.318	1.9996	1.9106	6.2436	6.2886	0.5568520
25	5.732	6.018	5.760	6.222	6.282	24.138	-0.299	2.0059	1.9201	6.2223	6.2820	0.5216320

Table 6: Relaxation Search, au=0.1 ap=0.1

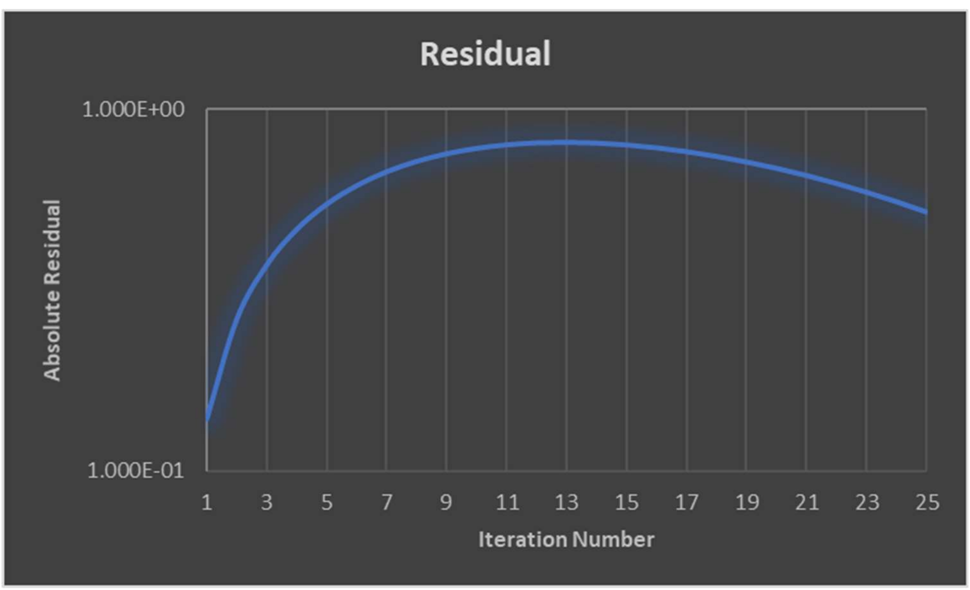

Figure 19: Residual Graph, Relaxation Search, au=0.1 ap=0.1

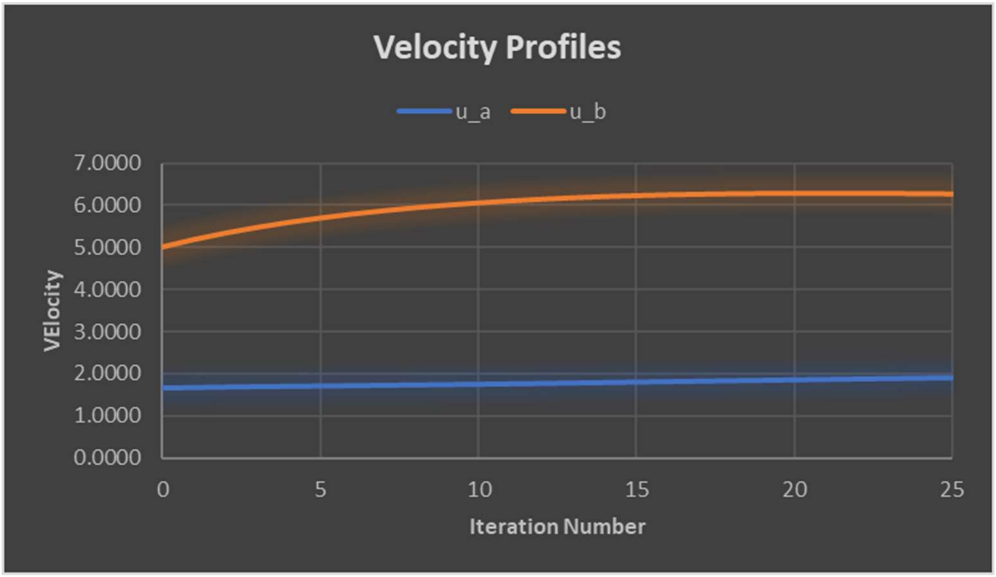

Figure 20: Velocity Profiles, Relaxation Search, au=0.1 ap=0.1

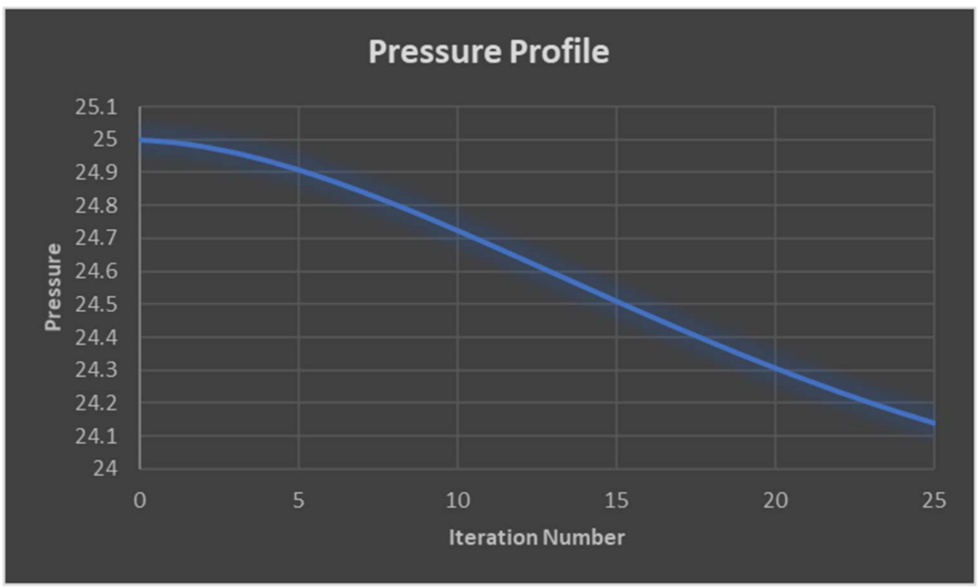

Figure 21: Pressure Profile, Relaxation Search, au=0.1 ap=0.1

Relaxation factors of 0.1 for both the velocity and pressure relaxation show a very poor convergence rate as seen in Figure 19. The velocity profile in Figure 20 shows a similar issue as the previous relaxation factor where at twenty-five iterations, the velocity does not meet the target velocity, though close. The pressure profile quickly falls to the target pressure but does not reach it quickly or at twenty-five iterations.

Using our knowledge of the effects of the relaxation factors on the convergence and stabilization, the relaxation factors of 0.5 look to be the best. With some guess and check, we can find the best combination of pressure and relaxation that results in a rapid convergence and stabilization of the mass residual, pressure profile, and velocity profile.

Relaxation Parameters				
au	0.6		ap	0.38

Iteration	F	F_A	F_A_R	F_B	F_B_R	p2_R	p'2	u_A	u_A_R	u_B	u_B_R	Residual
0	5	5	5	5	5	25	-	1.6667	1.6667	5.0000	5.0000	-
1	5.000	5.400	5.240	6.800	6.080	24.840	-0.420	1.8000	1.7467	6.8000	6.0800	0.8400000
2	5.240	5.427	5.352	6.549	6.362	24.639	-0.529	1.8089	1.7840	6.5495	6.3617	1.0096031
3	5.352	5.651	5.532	6.487	6.437	24.455	-0.485	1.8837	1.8439	6.4874	6.4371	0.9055801
4	5.532	5.768	5.673	6.344	6.381	24.306	-0.392	1.9225	1.8910	6.3435	6.3810	0.7078503
5	5.673	5.860	5.785	6.238	6.295	24.196	-0.289	1.9532	1.9283	6.2377	6.2950	0.5099799
6	5.785	5.917	5.864	6.155	6.211	24.120	-0.201	1.9724	1.9548	6.1550	6.2110	0.3466256
7	5.864	5.954	5.918	6.098	6.143	24.070	-0.132	1.9847	1.9728	6.0977	6.1430	0.2247806
8	5.918	5.976	5.953	6.059	6.093	24.039	-0.083	1.9921	1.9843	6.0592	6.0927	0.1397103
9	5.953	5.989	5.974	6.034	6.058	24.020	-0.050	1.9963	1.9915	6.0344	6.0577	0.0832239
10	5.974	5.996	5.987	6.019	6.034	24.009	-0.028	1.9985	1.9957	6.0190	6.0345	0.0472963
11	5.987	5.999	5.994	6.010	6.020	24.003	-0.015	1.9997	1.9981	6.0098	6.0197	0.0253631
12	5.994	6.001	5.998	6.005	6.011	24.001	-0.008	2.0002	1.9994	6.0046	6.0106	0.0125390
13	5.998	6.001	6.000	6.002	6.005	23.999	-0.003	2.0004	2.0000	6.0017	6.0053	0.0054079
14	6.000	6.001	6.001	6.000	6.002	23.999	-0.001	2.0004	2.0002	6.0003	6.0023	0.0016925
15	6.001	6.001	6.001	6.000	6.001	23.999	0.000	2.0003	2.0003	5.9997	6.0008	0.0000637
16	6.001	6.001	6.001	6.000	6.000	23.999	0.000	2.0002	2.0003	5.9995	6.0000	0.0007560
17	6.001	6.001	6.001	6.000	6.000	23.999	0.001	2.0002	2.0002	5.9995	5.9997	0.0009122
18	6.001	6.000	6.000	6.000	6.000	24.000	0.000	2.0001	2.0002	5.9996	5.9996	0.0008292
19	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0001	2.0001	5.9997	5.9997	0.0006636
20	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0001	2.0001	5.9998	5.9997	0.0004915
21	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	2.0000	5.9998	5.9998	0.0003442
22	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	2.0000	5.9999	5.9999	0.0002303
23	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	2.0000	5.9999	5.9999	0.0001480
24	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	2.0000	6.0000	5.9999	0.0000915
25	6.000	6.000	6.000	6.000	6.000	24.000	0.000	2.0000	2.0000	6.0000	6.0000	0.0000543

Table 7: Relaxation Search, au=0.6 ap=0.38

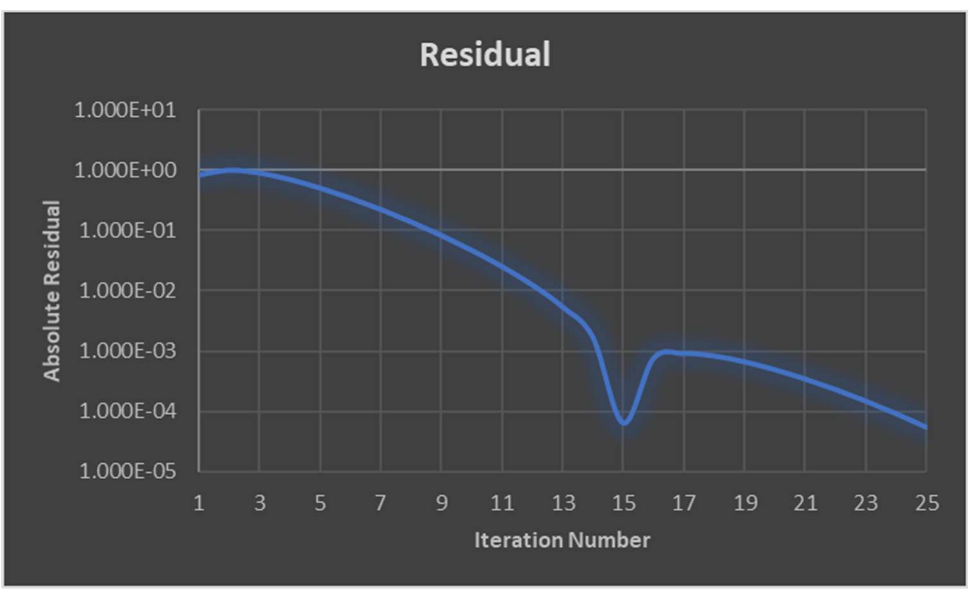

Figure 22: Residual Graph, Relaxation Search, au=0.6 ap=0.38

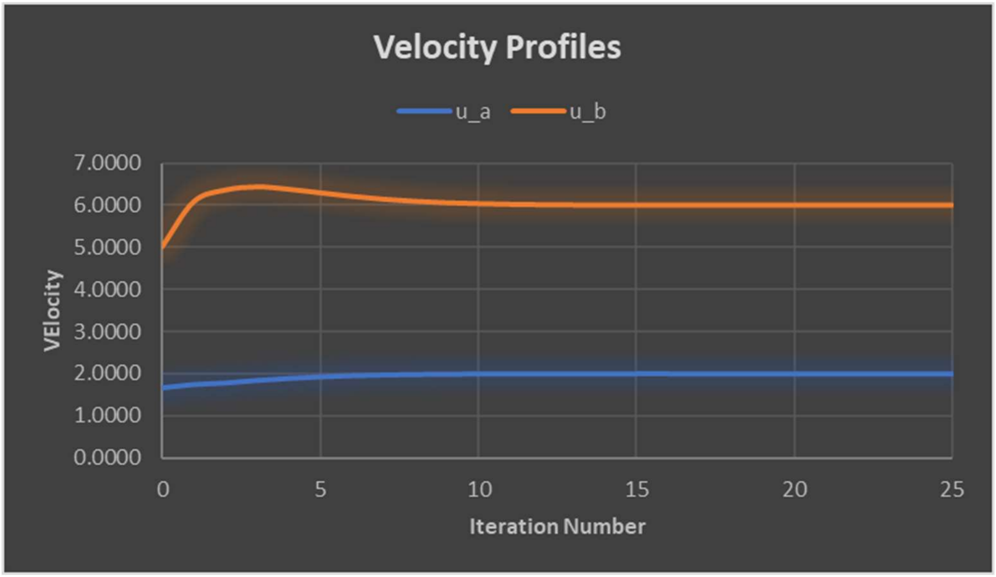

Figure 23: Velocity Profiles, Relaxation Search, au=0.6 ap=0.38

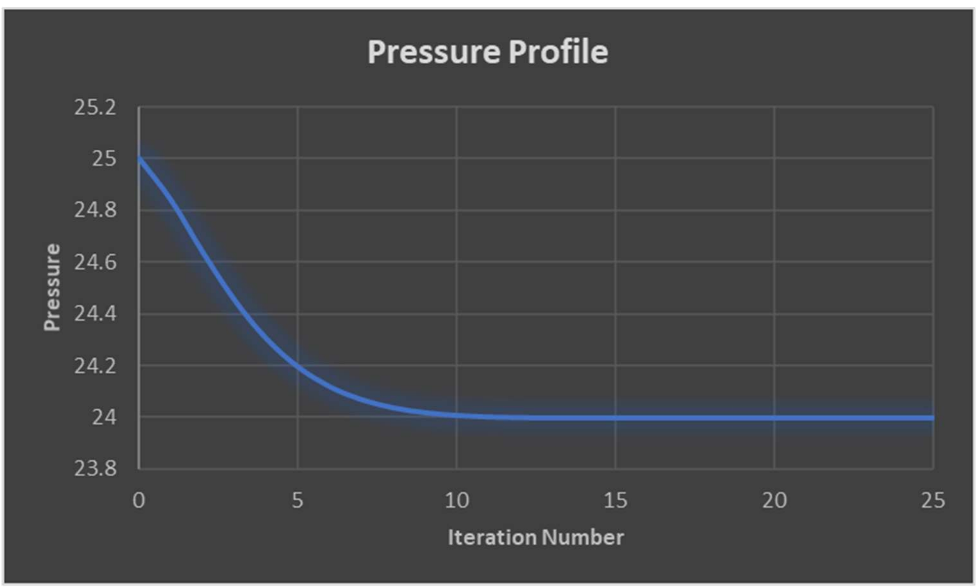

Figure 24: Pressure Profile, Relaxation Search, au=0.6 ap=0.38

With some exploration of the relaxation parameters, a combination that yields stable results with a high convergence rate is found to be au = 0.6 and ap = 0.38. For starters, the mass residual converges quickly and with minimal fluctuation (Figure 22). The velocity profile quickly reaches the target values of $\mathbf{u_A=2}$ and $\mathbf{u_B=6}$ and remains stable (Figure 23). The pressure profile can also be seen to reach its value of $\mathbf{P_2=24}$ very quickly and remains stable with not under or overshoot as seen with other relaxation parameters (Figure 24).

A model of the nozzle with this pressure and velocity field would be as pictured below.

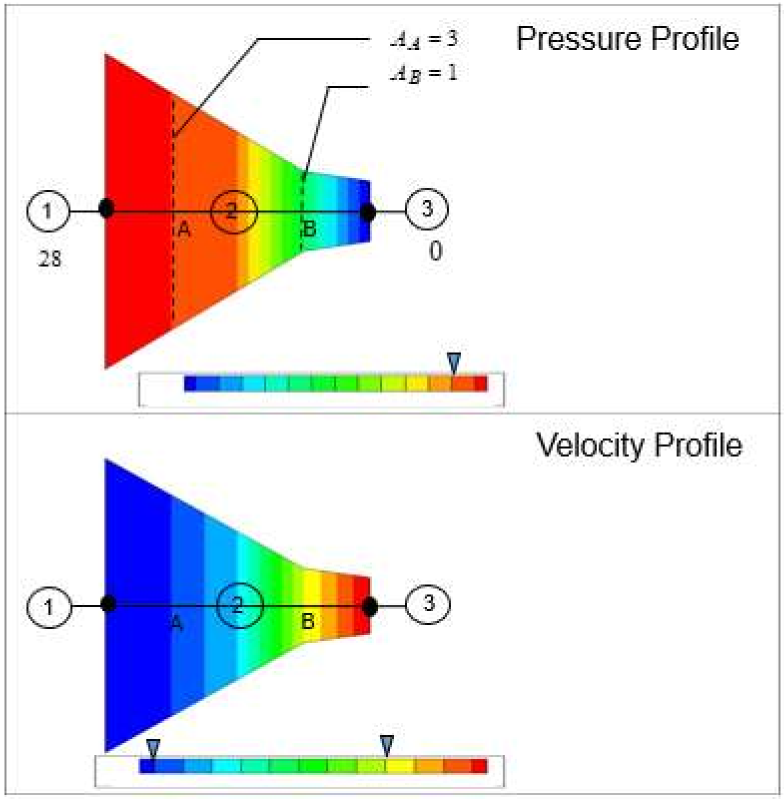

Figure 25: Velocity and Pressure Profiles

Discussion

Though the problem at hand is simplified by considering a steady, one-dimensional flow, it still demonstrates the iterative processes to define a computational fluid flow problem. The SIMPLE algorithm and relaxation methods utilized in this analysis can be used for more complex time dependent fluid flow problems that consider more dimensions. After obtaining the governing equations, iterative processes are very easy to implement in a successful way.

Analysis of any fluid dynamic problem involves the use of the conservation laws. Obtaining the governing equations for the problem allow us to study the behavior of the nozzle. From the continuity equation, momentum equation, and our boundary conditions, we can obtain the governing equations to fully describe our problem. Since these are very difficult to solve, we use the discretization method of finite difference to transform the PDE into algebraic form. This makes the solving easy to compute.

Also making the problem easier to solve is the SIMPLE algorithm. This is extensively used to solve the Navier-Stokes equations. This method can aid in solving for the pressure and velocity fields using an iterative process that completes once the continuity equation is satisfied, shown by the mass residual approaching zero. There are other methods that use a similar method. One method is a revised SIMPLE algorithm known as SIMPLER. This algorithm a discretized equation of continuity is used to find a discretized pressure equation rather than a pressure correction. The benefit to this method is that the pressure field is found without the use of a correction, though the velocity filed is still obtained using a correction like the SIMPLE method. Though the SIMPLER method is an option, we still choose to stick with the SIMPLE technique since usually in the SIMPLER method, we start off with a guess of the velocity, then solve for the velocity field without the pressure.

Consideration when solving computational fluid dynamic problems is the amount of time/money is used to obtain the solution as well as the accuracy. Depending on the problem, the solution could take many iterations to solve. This situation is not ideal. The faster a solution can converge; the less money and time will be used. Though a quick rate of convergence is idyllic, there may be issues with stability of the solution which in turn could affect the accuracy. This is seen in the initial calculation of the problem. We obtain a high rated convergence of the mass residual and pressure profile, but the velocity is seen to be unstable. Since we are trying to find the velocity at certain points, it is inaccurate to take a value with oscillating values. In order to find a balance between the convergence and stability of a solution, we can use relaxation factors.

Relaxation factors are used in computational fluid dynamic problems to determine the rate of convergence and stability of solutions. Relaxation factors can be “under-relaxed” or “over-relaxed”. An under-relaxed factor is when the factor is less than one, while above one would be an over-relaxed factor. The use of relaxation factors can reduce or increase the amount of iterations needed for a solution to be solved which can lead to either shorter or longer computational memory and time. The less memory and the quicker a solution can stabilize and converge, the better. When having instability issues, the solutions tend to overshoot or not hit the target solutions. In these instances, we can use a relaxation factor below one. When a solution is very stable, but convergence is slow, then an over-relaxation factor should be used.

A similar ideology would be PID tuning of controllers. The purpose of using pressure, mass flow, or temperature controller would be to set a target value. Taking a pressure controller as an example, setting the pressure target to be 90 psia and applying a pressure at the inlet of 110 psia would trigger the valve in the controller to close to restrict the amount of pressure coming in. Many times, the target value of 90 psia is overshot and takes a long time to reach or the target value is oscillated around. PID tuning allows a user to adjust the values of P, I, and D (which all effect the overshoot, time, and overall stability) to obtain a stable solution quickly.

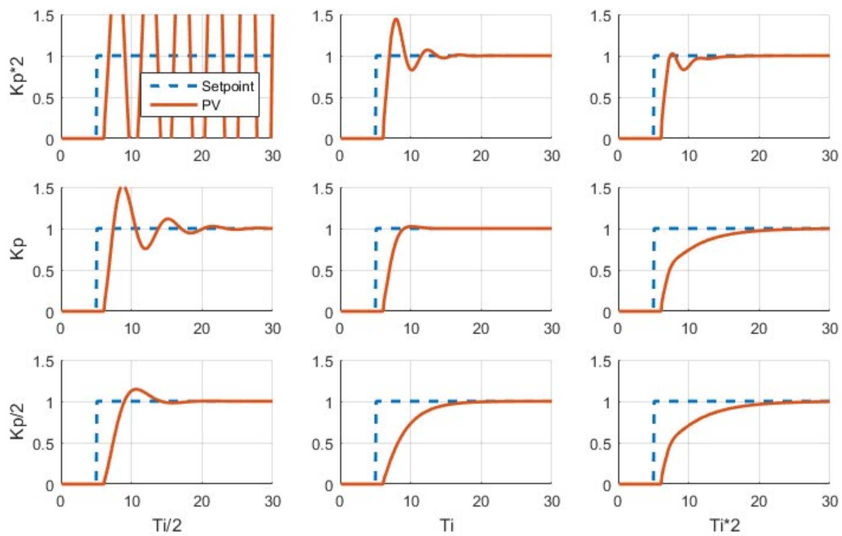

Figure 25: Stabilization Graphs to Set Points

Overall, the solution approach to this problem is seen to be appropriate. Using the SIMPLE algorithm in conjunction with relaxation factors, we found a balance between the rate of convergence and the stability of the solution. To achieve this balance, relaxation factors of $\alpha_u = 0.6$ and $\alpha_p = 0.38$ were used. Depending on the goals of the problem set (accuracy and time), the relaxation factors could be adjusted. Using a guess and check, the relaxation factors used for the final solution were found to be the best, but there may be a more ideal solution. With these relaxation factors, we were able to converge quickly and also obtain stable solutions for the points of interest in the nozzle.

References:

RPI Professor Frank Cunha Class Notes, PowerPoints, Video Lectures, and Video Meeting Conferences

Pletcher, Richard H., et al. *Computational Fluid Mechanics and Heat Transfer*. 3rd

RPI Student Meetings and Email Documentation. Approach Team. Ryan Quinn, Patrick Hart, Abraham Martinez, Jeremy Clavette, Stephen Zalys, and Mark Winters.