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Milestone 3 Report

Discrete Event Simulation of a Flow Metering Element Assembly Process

Prepared By: Ian Navin

Topic Expert: Professor Ernesto Gutierrez-Miravete

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Abstract

Industries across the globe are learning to evolve and adapt to the growing demand. Adopting lean and six sigma methodologies and principles can aid in eliminating waste and consequently increase efficiency and quality. Brainstorming ideas for change is one thing but gaining insight into how those changes will impact a random manufacturing process over time is nearly impossible without mathematical modeling and virtual simulation. Discrete Event Simulation sheds light on understanding these changes in a very short time period without having to implement changes in the real world. Many inputs and changes along the way can be simulated with ease. Using an M/M/1 queueing theory for understanding the conventional drilling process of a Flow Metering Assembly starts the building blocks to using DES to simulate the full assembly process while making changes to machines and people. With the M/M/1 queueing theory, I was able to find the steady state calculations based on the process inputs of mean arrival and process times. Using excel to perform a simple DES of just 10 components, I find that I was not at steady state conditions (maximum 89.9% difference). After simulating 1000 components 500 times, I was able to show that steady state was reached with a maximum of 1.10% difference. We can then move to a simulator software such as ProModel, but before we can rely on this software to build complex processes to simulate, we must verify and validate the simulator by comparing to the steady state calculations of M/M/1 theory, which we were able to do with a percent difference of all key performance indicators below 0.40% and the server utilization percent difference of 0.265%. With the single server validated, building the model up was assessed by comparing two servers to the single server where we see a 55% reduction of time in the system and 93% reduction of time in the queue. These building methods can be applied further to a larger assembly process of the metering element, and we now know that the results are reliable.

1.0 Introduction and Background

Industries around the world, from service to manufacturing, face an evolution of innovation and efficiency where the adaptation of everchanging technologies can determine future success or failure. Competitors in all industries are finding ways to edge out the competition while continually improving the quality of their end product and subsequently, their customer relationships. Lean principles, born from the manufacturing industry, has transformed industries in a way that promotes continuous improvement. Utilizing these principles to increase efficiency can impact many key performance indicators held by companies by reducing waste in all forms. The principles, as described by Womack and Jones' book titled, "The Machine That Changed the World", are shown in Figure 1.



Figure 1: The Five Lean Principles

As the goal of lean manufacturing is to reduce waste, the waste must be identified. In lean practice, eight types of waste exist: Defects, Over-Production, Waiting, Unused Talent, Transportation, Inventory, Motion, and Extra-Processing.

When viewing company improvement from a project management perspective, one can utilize Six Sigma methodology. The purpose of this methodology is to reduce the defects and variations in a process using statistical analysis. The approach to accomplishing this goal is by using what is known as a DMAIC method, which is an acronym for: Define, Measure, Analyze, Improve, and Control. As visual representation can be seen in Figure 2.



Figure 2: Six Sigma: DMAIC Improvement Process

By combining the ideas of Lean Manufacturing and Six Sigma, one will be utilizing Lean Six Sigma. Since the two methods complement each other, they can be meshed to reduce process defects and waste while giving a structure to the approach.

Many tools exist to implement the Lean Six Sigma ideology. The time dedication of a DMAIC project will depend on the size of the project scope. Some projects can yield improvements through many basic tools, but in the manufacturing environment, with many moving parts, implementation of changes and understanding of performance can depend on many factors and unknowns. To aid in this difficulty, mathematic modeling of a manufacturing system was developed to predict how a real system would behave overtime. Not only do these mathematic models give meaningful insight into complex systems, but they are also financially advantageous. Models that can investigate the performance of manufacturing systems include Optimization Models and Stochastic or Queueing Models. To ensure that the manufacturing system analysis is performed correctly, Buzacott and Shanthikumar suggest the following steps:

1. Identify the issue or issues
2. Gather all the relevant facts and information
3. Select a modeling method
4. Develop, test, verify, and validate the model
5. Carry out production runs of the model
6. Clearly communicate the results of the model to others

In the process of developing, testing, verifying, and validating a model, Discrete Event Simulation (DES) can be used. DES is a simulation method that views a system at discrete moments in time. These discrete moments can be defined as a certain state of a system. Viewing these states as impacted by random effects can ultimately make accurate predictions of how a real-world system will change and adapt to small or large variability changes to the inputs. This is achieved by predicting long term system performance, showing interactions among the components and the

operators, tracking statistics to calculate performance metrics, accurately modeling randomness, and compressing the time to find the outputs. With the ability to virtually evaluate changes to the system, companies can make decisions more quickly and effectively.

Some limitations to note about DES is that it is only a tool for use of evaluation, not optimization. For the latter, a design of experiments should be run. Depending on the inputs and size of the system, a DES may not be necessary as the evaluation can be calculated using first order iterations.

The way a DES is approached will ultimately decide its' success. According to the presentation, "Introduction to Simulation and its Benefits" presented by Edward J. Williams, the nine steps for success in these simulations comprise of:

1. Define the Opportunity
2. Identify Objectives/Scope
3. Collect Data
4. Build Model
5. Verify/Validate
6. Experiment
7. Analyze Results
8. Implement Results
9. Document

2.0 Problem Statement

In an attempt to study, create, develop, test, verify, and validate a real manufacturing process/system from a logistics standpoint, I chose to focus on Discrete Event Simulation of a Flow Metering Element Assembly Process. This process follows a batch processing mode in a process layout manufacturing system meaning that the products will follow a sequential pattern of workstations or operations on the production floor in accordance with the job book routing. The batch processing is utilized to ensure that there is a high level of lot traceability.

The Flow Metering Element Assembly process relies heavily on the operation focused on the metering orifice size. With a multiple orifice component, a single metering element could potentially have hundreds or thousands of metering orifices in series. The multiple orifice element can be seen in Figure 3 sandwiched between two safety screens that protect the orifices from any contamination. The element can vary in length depending on how restrictive the desired component is or how much the customer would like the component to flow fluid at a certain pressure and temperature.

To produce a flow metering element with a high quantity of metering orifices and a high quality of metering orifice size, the machine producing the orifices is critical. One process that is currently used to manufacture the metering orifices holes is a conventional CNC drilling station. Another process that the department is in the process of implementing is a laser drilling machine. Though the cycle time of each type of machine is similar, it is known that the laser can produce much more consistent, accurate, and custom holes when compared to the conventional drilling. The laser drilled holes also require much less post-processing time than the conventional drilling machine holes.



Figure 3: Multi-Orifice Flow Metering Element with Safety Screens

Of interest is the evaluation of the different processing methods and varying inputs to these methods: Conventional Drilling, Laser Drilling, and the Combination of Conventional and Laser Drilling. The process flow map can be seen below in Figure 4 for all methods.

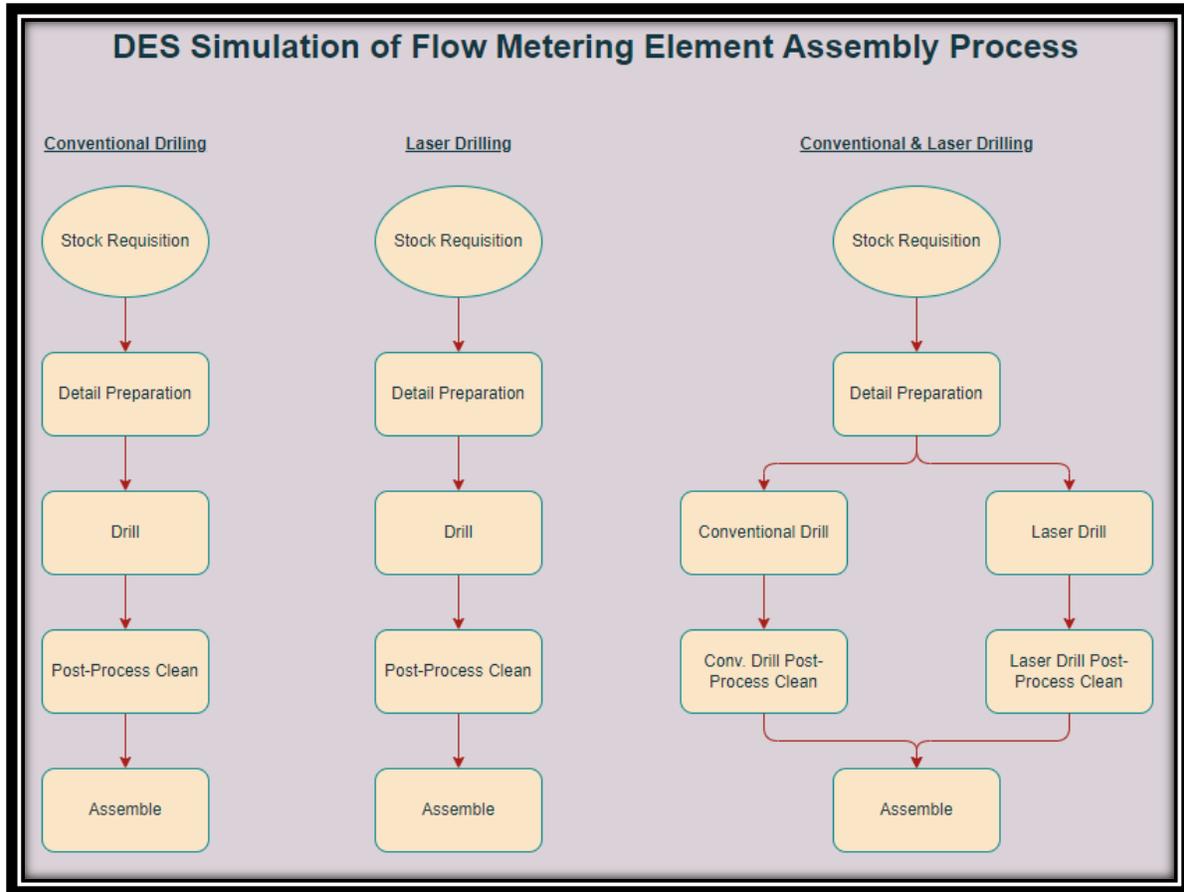


Figure 4: Process-Flow Map: Flow Metering Element Assembly

As with many simulations we will start off with a first order iteration of this problem for our baseline. To aid in the simplicity of the system and calculations, we will evaluate the one of the process steps with a queuing theory method known as M/M/1. This type of queuing method comprises of a single server, operator, or machine that processes one unit at a time. Any units arriving while the server, operator, or machine is busy with another unit, wait in a queuing line that can be infinitely long. Both the inter-arrival and processing times are assumed to be exponentially distributed since this distribution is used for all interarrival situations.

We can then utilize discrete event simulation software to build the M/M/1 queue into a model and verify and validate that the results match the queuing theory preliminary scoping calculations. With the model validated, we can move onto refining the modeling skills to produce multiple machines at operations which will then build to replicating models as shown in Figure 4. Curating accurate processing and input data will be essential to the accuracy of the model outputs. After modeling the current state and validating with real data, we can look at ways to improve the process.

3.0 Analysis:

3.1 Steady State M/M/1 Queueing Theory: Preliminary Scoping Calculations

As stated in the problem statement, we will perform the preliminary calculations for an M/M/1 queueing system which will yield key performance parameters of the system at steady state conditions. These key performance parameters include:

L – The Total Number of Components in the System

L_q – The Total Number of Components in the Queue

W – The Average Time Each Component Spends in the System

W_q – The Average Time Each Component Spends in the Queue

Since performing a DES is evaluating a system one discrete event at a time, one must always ensure that they finish evaluating the system once it become steady state. Failure to do so will result in inaccurate results to base decisions from. The M/M/1 queueing theory has derived equations that show steady state performance parameters that can be used to determine if the DES has reached steady state. Figure 5 shows a simplification of the process to be evaluated for use in the M/M/1 queueing system.



Figure 5: Baseline Simulation Process Flow

For the problem to be solved, we need a list of inputs which will consist of exponentially distributed inter-arrival and processing times, number of machines, and number of operators. From the inter-arrival time, we can obtain the arrival rate of components, λ . Similarly, we can obtain the processing rate of components, μ , from the processing time. When considering the exponentially distributed parameters, we use a mean of $\frac{1}{\lambda}$ and $\frac{1}{\mu}$. Process inputs for the baseline simulation can be seen in Table 1 below.

Baseline Simulation M/M/1 Process Inputs	
Machine	Servotronic CNC Drill Press
Number of Machines	1
Number of Operators	1
Inter-Arrival Time of Components	2.08 Minutes
Mean Arrival Rate of Components to Visco Dept (λ)	1 Component/2.08 Minutes
Processing Time of Components at Machine	1.25 Minutes
Mean Processing Rate of Discs (μ)	1 Component/1.25 Minutes

Table 1: Process Inputs - M/M/1 Queueing System

These process inputs have been verified as accurate and reliable for the process under investigation. There are times where these values deviate for different product lines, but the inter-arrival and processing times have been chosen for components that represent the mean. The use of the exponentially distributed values allows the randomness of the process to be represented.

In collaboration with Figure 5, Figure 6 shows a visual of the single server queueing system.

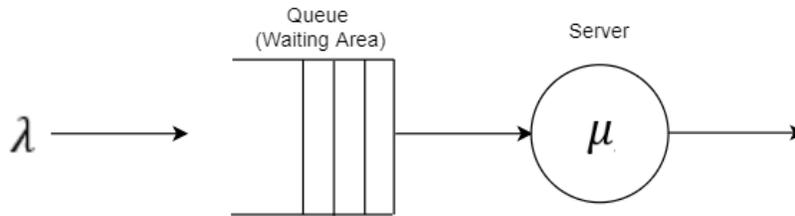


Figure 6: Single Server Queueing System

With one server in the queueing system, we can also evaluate the utilization of the server, ρ . This value can be determined by the ratio of the mean arrival rate of components and the mean processing rate

$$\rho = \frac{\lambda}{\mu}$$

(1)

For our process inputs, the server utilization comes out to

$$\rho = \frac{\lambda}{\mu} = \frac{\frac{1}{2.08}}{\frac{1}{1.25}} = \frac{0.4807}{0.80} = \mathbf{0.601}$$

Evaluation of the key performance indicators can be conducted using the following derived equations for an M/M/1 system at steady state

$$L = \frac{\lambda}{\mu - \lambda} = \frac{\rho}{1 - \rho} \quad (2)$$

$$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)} = \frac{\rho^2}{1 - \rho} \quad (3)$$

$$W = \frac{1}{\mu - \lambda} = \frac{1}{\mu(1 - \rho)} \quad (4)$$

$$W_q = \frac{\lambda}{\mu(\mu - \lambda)} = \frac{\rho}{\mu(1 - \rho)} \quad (5)$$

Using Equation 2 through Equation 5 in conjunction with the process inputs, the M/M/1 Steady State Calculations are as follows

M/M/1 Steady State Calculations	
<i>L</i>	1.51
<i>L_q</i>	0.91
<i>W</i>	3.13
<i>W_q</i>	1.88

Table 2: M/M/1 Steady State Calculation KPI's

3.2 Discrete Event Simulation Study of a Single Server Queue Using Excel

Many software packages exist to perform DES studies (ProModel), but some simple simulations can be conducted in a software such as Microsoft Excel. To start, we know that our manufacturing process under evaluation is stochastic and random in nature. This is due to the complexities of a manufacturing process with many factors affecting the inter-arrival and process times (i.e. machine down time, etc.). As discussed for the M/M/1 queueing system, the randomness will be shown through an exponential distribution.

To start off the evaluation, the excel spreadsheet will initially be set up to look at a discrete event simulation of the first ten components. The random function in excel, “=Rand()”, will output a uniformly distributed random number between zero and one for each component. To convert this number to an exponentially distributed random number for the inter-arrival time and the process time, we can use the following equation:

$$\mathbf{Inter - Arrival Time}_{expon\ dist} = \frac{-\ln(1 - \mathbf{Random}_{unif\ dist})}{\lambda} \quad (6)$$

$$\mathbf{Process Time}_{expon\ dist} = \frac{-\ln(1 - \mathbf{Random}_{unif\ dist})}{\lambda} \quad (7)$$

Once the exponentially distributed random numbers are assigned to each component for the inter-arrival time and process time, the arrival times and service start times can begin for the first component. For the next components, an if statement will determine whether the component can be processed or if it must wait in queue for the component before to complete. The amount of time processing the components, time waiting, total time in the system, and even the idle time of the machine can all be calculated for each component. An example of the ten-component system using the same process inputs as the baseline steady state calculations can be found in the Appendix. The key performance indicators for this system are shown in Table 3.

Initial DES Simulation Outputs	
<i>L</i>	2.98
<i>L_q</i>	2.38
<i>W</i>	6.20
<i>W_q</i>	4.954

Table 3: Initial 10-Component DES Simulation KPI's

Comparing the KPI's with the steady state calculation in Table 4, we can see a large discrepancy.

M/M/1 Steady State Calculations		10 Component DES	Percent Difference
L	1.51	2.98	65.47%
L_q	0.91	2.38	89.36%
W	3.13	6.20	65.81%
W_q	1.88	4.954	89.96%

Table 4: Steady State vs 10 Component DES KPI's

Increasing the number of components (i.e. 1,000) can increase the chances of becoming steady state, but we can also create a Data Table from the What-If analysis function in excel to essentially run the 1,000 component event as many times as we desire. The simulation should be run until we achieve steady state values.

For this example, we will use a 1,000-component study and compose a Data Table of 50 columns and 10 rows to yield 500 different studies of the 1,000-components (see Appendix). Taking the average expected wait time in queue of all of the studies and using this to calculate the KPI's yields results as displayed in Table 5.

1,000 Comp → 500 Study DES Simulation Outputs	
L	1.50
L_q	0.90
W	3.12
W_q	1.87

Table 5: Larger DES Simulation to Approach Steady State

Comparing these KPI's to the steady state KPI's yields

M/M/1 Steady State Calculations		1K Comp, 500 Study Avg DES	Percent Difference
L	1.51	1.50	0.66%
L_q	0.91	0.90	1.10%
W	3.13	3.12	0.32%
W_q	1.88	1.87	0.53%

Table 6: Steady State vs 1K Component 500 Study Avg DES KPI's

3.3 First Order DES Study of a Single Server Queue Using ProModel

As discussed at the beginning of section 3.2, there are software packages that exist that are able to simulate such a simple model. The software is very sophisticated in the move logic and the logic to integrate at different locations of queues. Many calculations and checks go into the excel calculations and when trying to reach steady state, many instances have to be iterated. To show the ability of ProModel and to verify the use and setup of the simulation, a simple single server queue was setup in the software and the simulation run using the same input parameters in Table 1.

Setup in ProModel starts by identifying the locations in the model; in this case we have an arrival queue and a drill press acting as the single server location. The queue is set to have an infinite capacity while the drill press is only able to process one component at a time. The decision rule for the queue is prioritizing the oldest component and also a first in first out queueing output. The drill press also prioritizes the oldest component but has no queueing capability. The locations in the simulation layout can be seen in Figure 7 below.

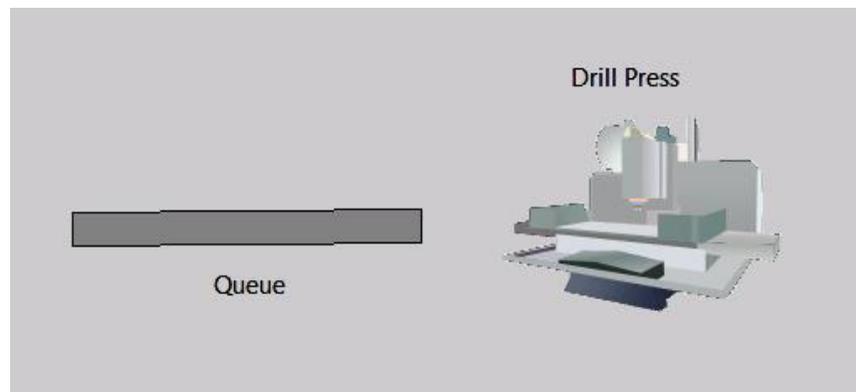


Figure 7: Single Server Queue Layout – ProModel

Next, the entity to be processed is assigned. In this instance, we use a gear as a representation of a fret since we don't have a picture of the fret in the simulation. We show the gear arriving with an exponential distribution of one component every 2.08 minutes as defined in Table 1.

The ability of the drill press to process the entity is the next step which requires operation commands for the entity at each location. For the drill press location, the exponential distribution of the processing time is used by inputting 1 component every 1.25 minutes. The way the components move is conducted through the routing where the destinations per the process map can be made, and we also use rules and move logic to depict the simulation most accurately. The entities in queue for the single server are moved to the drill press and from the drill press, routed to the exit.

The logic in the arrivals and the move logic during processing can be useful for viewing variables live on the simulation screen. For this simulation, I created attributes and variables that watch the Total Time in Queue, the Total Time in System, Component Number, Average Time in System, and the Average Time in Queue. The only downfall is that to view these numbers live, the animation cannot be disabled, so the simulation time took much longer.

The simulation run time was set initially for 100 hours, but the simulation did not reach steady state. To ensure steady state was reached, a run time was set for 5,000 hours and was observed that steady state was actually reached around 3,000 hours, though to get to around 95% of steady state, the simulation took 900 hours. The values obtained are shown in Figure 8.

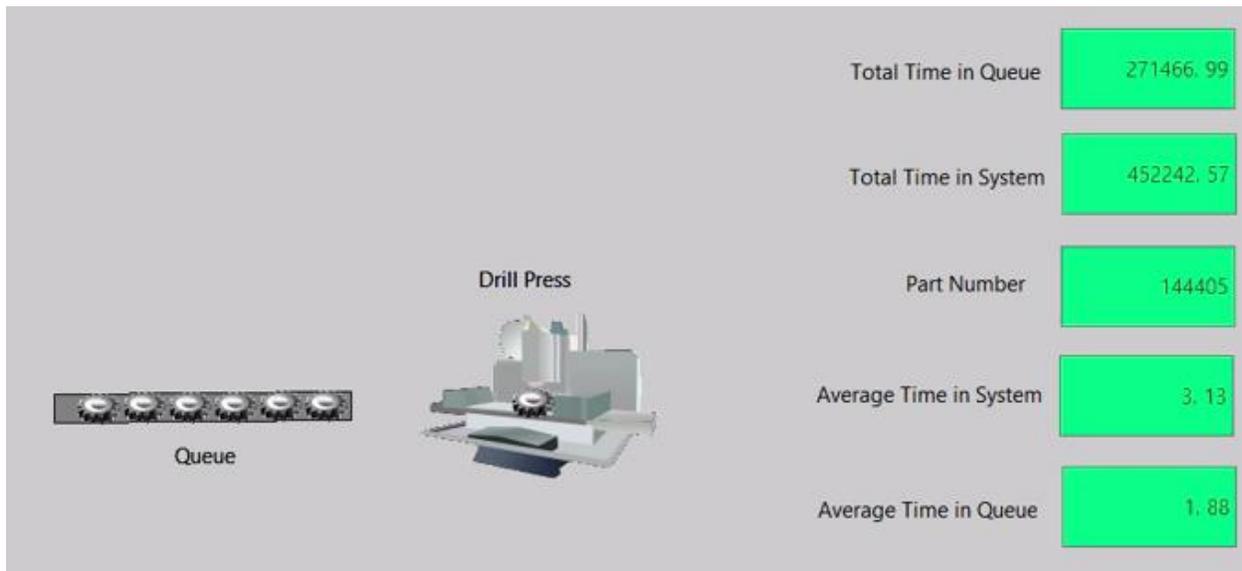


Figure 8: Single Server Simulation – ProModel

It can be seen that steady state was reached since the performance metrics of Average Time in System and Average Time in Queue match the values calculated in the preliminary scoping calculations for steady state M/M/1 queue. This verifies and validates that the model was built correctly with the correct logic.

After the simulation is completed, we can view the results of the simulation in a different way using the Output Viewer. The scoreboard shows similar statistics as the attributes and variables that show during the simulation in Figure 8. Figure 9 shows the scoreboard for this simulation. We can see that from the total exits of 144,405 and the total time in queue of 271,466.99, the average time in queue is 1.88 ($271466.99/144,405$) which matches the average time in queue from Table 2.

Scoreboard				
Name	Total Exits	Average Time In System (Min)	Average Time In Operation (Min)	Average Cost
Gear	144,405.00	3.13	1.25	0.00

Figure 9: Single Server Model Scoreboard

Another way to see if our model can be validated is by looking at the drill press utilization from the simulation and compare to the preliminary scoping calculations. After using equation 1, we ended up with a server utilization of 0.601 or 60.1%. The graphic below, Figure 10, shows the drill press utilization from the simulation as 60.26%.

To use the utilization even further, we can use equations 2 and 3 to find the total number of components in the system and queue for this simulation as shown in Table 7.

M/M/1 Steady State Calculations		ProModel Simulation	Percent Difference
L	1.51	1.516	0.397%
L_q	0.91	0.913	0.329%
W	3.13	3.13	0%
W_q	1.88	1.88	0%

Table 7: M/M/1 Steady State vs ProModel Single Server



Figure 10: Drill Press Utilization (Single Server)

Calculation of the percent difference can be seen as:

$$\frac{|.601 - .6026|}{.6026} * 100 = 0.265\%$$

This shows that the experimental value from the simulation is within 0.265% of the calculated value from the preliminary steady state calculations which further validates the model.

The output viewer also displays the state of the entity; whether it is waiting, in operation, blocked, or in the move logic. We can see from Figure 11 that the component is never in move logic and is in operation almost 40% of the time while waiting for 36% of time and blocked the other 24%.

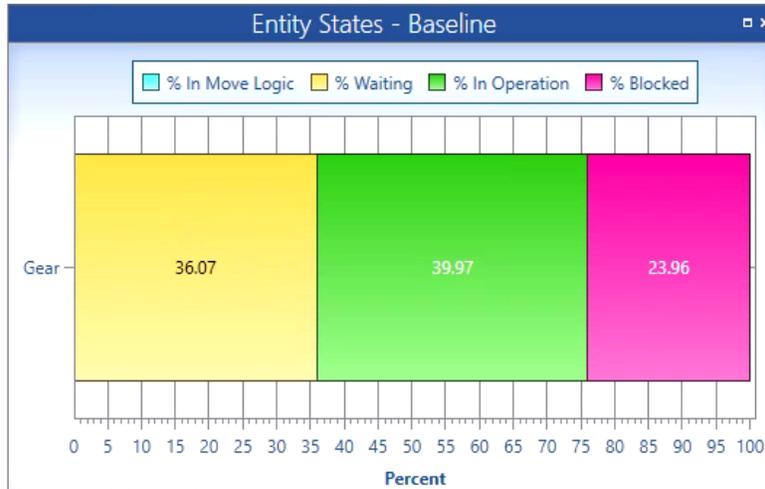


Figure 11: Entity States - Single Server

The waiting time accounts for any time that the entity is waiting in queue behind a blocked entity or just in the queue in general whereas the blocked time shows the time the entity spends waiting for a location to have capacity in order to be processed.

3.4 First Order DES Study with Multiple Servers in ProModel

With the information found from the single server study, we have verified and validated that the simulation model is a valid tool that will produce accurate answers. This leads us to use the software to build upon our model to higher complexities. To slowly build the model, we can investigate using two servers where the components in queue will go to the first available server to try and gain the most utilization.

There are a few ways to build this model where first we can build upon the single server and add in a new drill press in the locations. We set everything the same except we ensure that the first available server takes the available entity. This simulation was run first, and the results established, but to make the simulation easier, there is a selection for a location identity to contain several units. In this case, the drill press location was set to two units as seen in Figure 12. This method is more robust as it ensures no mistakes are made when trying to duplicate the location in all the processing and move logic. Both methods were run, and we found the same results. These results can be seen in Figure 13.

Icon	Name	Cap.	Units
	Queue	INFINITE	1
	Drill_Press	1	2
	Drill_Press.1	1	1
	Drill_Press.2	1	1

Figure 12: Drill Press Location Units

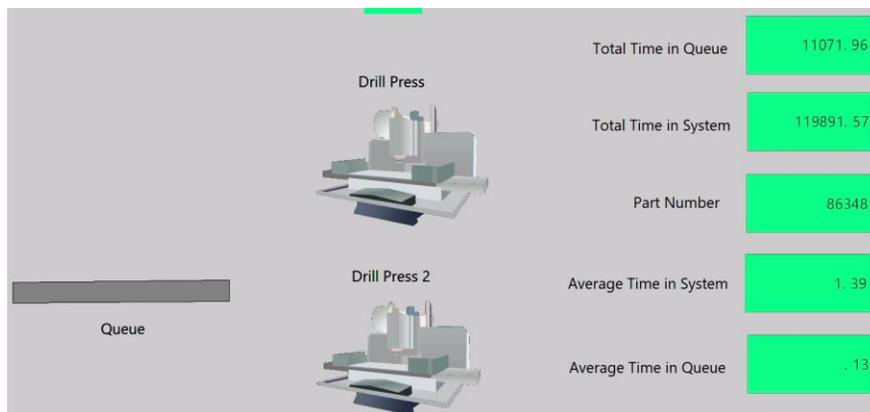


Figure 13: Two Server Simulation (ProModel)

The simulation was run for the same 3,000 hours as the single server to show the differences, though it was observed during the simulation that steady state was reached after 900 hours. We can see from the initial animated results as described by the variables and attributes in Figure 13 that the Average Time in the System is 1.39 minutes, and the Average Time in Queue is 0.13

minutes. This time reductions can be expected as having multiple stations to process the components will improve the performance metrics. The drill press utilization can also be seen in Figure 14.

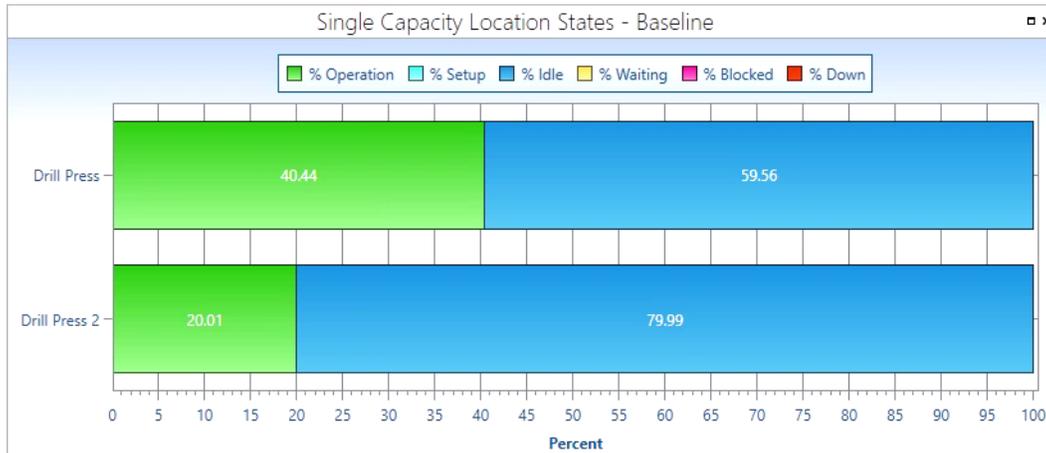


Figure 14: Drill Press Utilization by Location

The first drill press was in operation for 40.44% of the time while the second drill press was in operation for 20.01% of the time. This totals to 60.45% drill press utilization for the same amount of time that we simulated the single server which had a utilization of 60.26%. We see for the multiple servers that there is a high percentage of idle time.

A large improvement is seen in the entity state as shown in Figure 15 where the entities are in operation 90.77% of the time while waiting 2.84% of the time and blocked only 6.4% of the time.

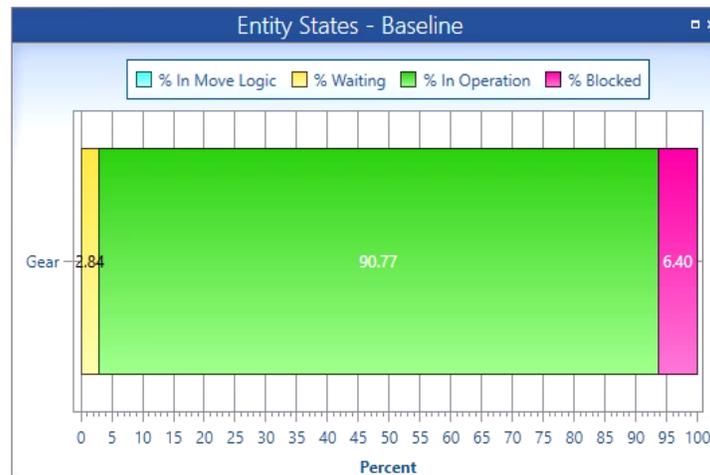


Figure 15: Two Server Entity State

We also see that with the interarrival times, and the processing times shown with two drill presses that the queue is empty most of the time (95.74%), so as components come in, they have

an opportunity to exit the queue immediately because of the options of servers. This can be shown in Figure 16.

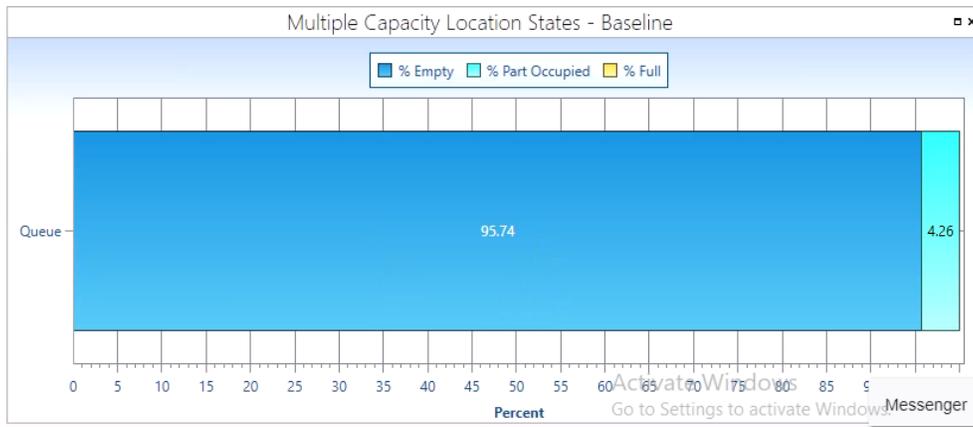


Figure 16: Queue State - Two Servers

We know that the old performance metrics for the number of components in the system will increase a bit since more components can be processed at a time but looking at the average time in the system and the time in queue, we see from Table 8 that we have a 55.6% reduction of time a component spends in the system and a 93.1% reduction in time spent in the queue.

ProModel Single Server	ProModel Two Servers	Percent Difference	
W	3.13	1.39	55.6%
W_q	1.88	0.13	93.1%

Table 8: Single Server vs Two Server Simulation

3.5 DES of Conventional Drilling and Laser Drilling Processes in ProModel

With our first order model in ProModel validated with the preliminary scoping calculations and our cultivation of skills added machines and locations to the simulation, we can now analyze more realistic systems. The problem statement wishes to explore the differences in using these methods separately and together to monitor manufacturing efficiency.

First, we curate the data pulled from five years of data in the Visco department. Since jobs are batch processed, it is important to curate the data on a per job basis, not on per part basis. Using this methodology, we use the most common job quantity for standard product of 315 pieces. The interarrival time is the same for conventional and laser drilling as they are part of the same department. Conventional drilling processing times can be found in Table 9 and laser drilling in Table 11. Table 10 gives a small breakdown of the increase time given to the post-processing of the conventional drilling process.

Manual Drilling			
Job = 315 pcs			
Process	sec/part	min/part	min/job
Arrival	16.000	0.267	84.000
Detail Prep	9.000	0.150	47.250
Drilling	75.000	1.250	393.750
Post Processing	29.567	0.493	155.225
Assembly	14.280	0.238	74.970

Table 9: Conventional Drilling Inputs

Conventional Drilling Post - Process Break Down			
Post Processing	min/job	min/part	sec/part
simple green	50.000	0.159	9.524
sanding	37.500	0.119	7.143
degreaser	67.725	0.215	12.900
Total	155.225	0.493	29.567

Table 10: Conventional Drilling Post-Processing

Laser Drilling			
Job = 315 pcs			
Processing	sec/part	min/part	min/job
Arrival	16.000	0.267	84.000
Detail Prep	9.000	0.150	47.250
Drilling	67.500	1.125	354.375
Post Processing	5.714	0.095	30.000
Assembly	14.280	0.238	74.970

Table 11: Laser Drilling Inputs

To not only look at performance metrics of jobs in the system, but of the cost analysis, we collected labor rates for the facilities that the parts are processed as seen in Table 12.

Cost Analysis Info	
Facility	Cost
RA008	\$ 80.30
RL002	\$ 82.42

Table 12: Labor Rates for Processing Facilities

Each new location will be added to the simulation in a singular manner to slowly build the model and compare the laser to the conventional drilling. The locations will be the same, but the processing times will differ as will the labor rates where some of the tasks are performed. The locations will follow the flow chart shown in Figure 4.

With many more location introductions, there will be statistical variability at each location that needs to be accounted for. Many distributions exist that can aid in predictions and when they are utilized is important. One could use deterministic values, but this does not account for the variability that could occur in a simulation over many hours. These distributions are outlined in Reference 5 and are as follows:

- **Binomial.** Useful when there are only two possible outcomes of an experiment which is repeated multiple times.
- **Geometric.** Useful also when there are only two possible outcomes of an experiment which is repeated multiple times.
- **Poisson.** Useful to represent the number of incoming customers or requests into a
- **Normal.** Useful to represent the distribution of errors of all kinds.
- **Lognormal.** Useful for representation of times required to perform a given task or accomplish a certain goal.
- **Exponential.** Useful to represent interarrival times in all kinds of situations.
- **Gamma.** Useful also for representation of times required to perform a given task or accomplish a certain goal but more general.
- **Beta.** Useful as a rough model under situation of ignorance and/or to represent the proportion of non-conforming items in a set.
- **Erlang.** Useful to represent systems making simultaneous requests for attention from a server.
- **Weibull.** Useful to represent the life and/or reliability of components.
- **Uniform.** Useful when one knows nothing about the system.
- **Triangular.** Useful when one knows little about the system.

The exponential distribution was utilized in the preliminary scoping calculations and the first order models, but now should only be used for the interarrival time of the jobs as described in the list above. To show the differences in using differing distributions that may lend themselves

well to this simulation, we implement the distributions at the locations to compare. Many of these distributions require the mean value and the standard deviation. For these realistic models, we were able to curate the standard deviations to be approximately 10% of the average processing times. The distribution analysis of the Deterministic, normal, exponential, and lognormal distributions is shown in graphical format in Figure 17 for a one working year analysis.

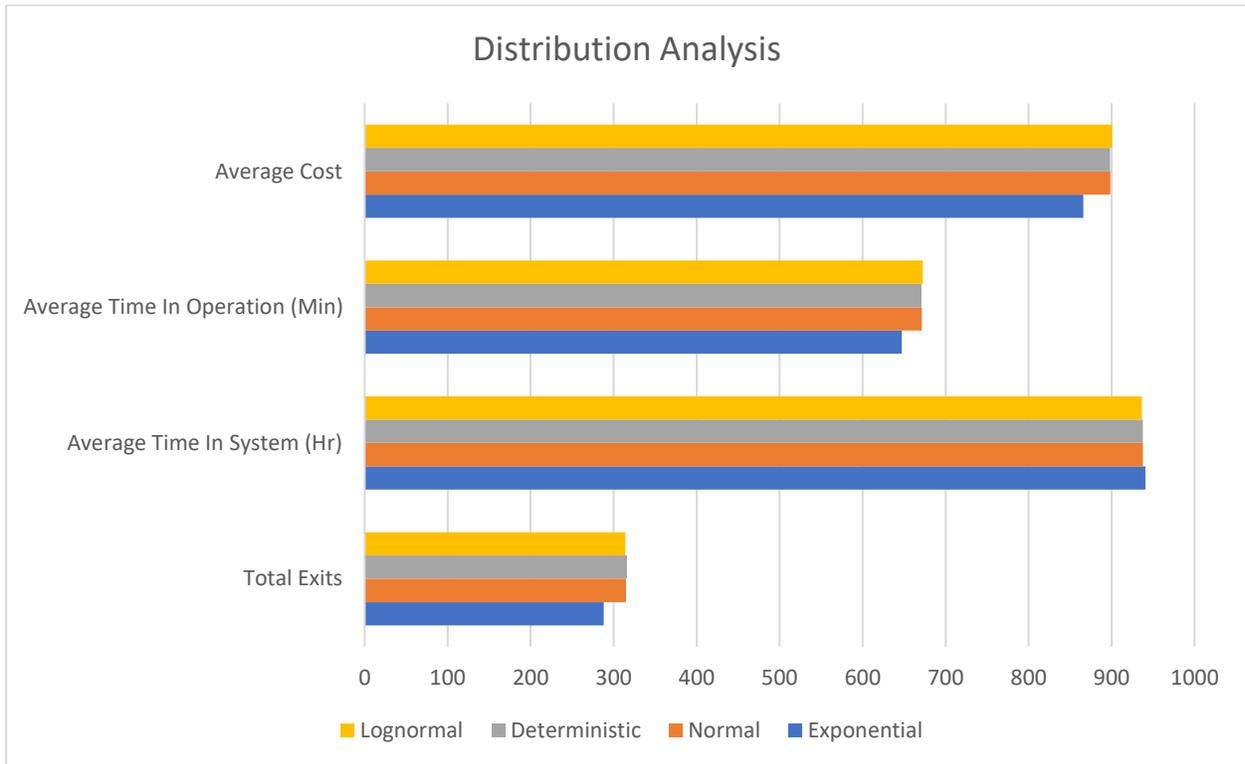


Figure 17: Distribution Analysis (One Year)

We see that the exponential distribution used throughout varies from each parameter. The other distributions vary a bit too, but not nearly as much. In queuing systems, the lognormal distribution seems like a good selection since each station is performing a task. The normal distribution is used for a queueing system that is fairly constant but with some random variability, which describes the process very well. A normal distribution also lends itself to our reliability and maintainability since the normal distribution is best at predicting failures due to wear¹³, which is what we experience with our tools and the material we cut through.

Using the normal distribution for each of the processing times at each step, we can simulate the conventional drilling with one machine for each location and simulate for one working year which equates to 2,080 hours. The scoreboard results are as shown in Figure 18.

Scoreboard

Name	Total Exits	Average Time In System (Min)	Average Time In Operation (Min)	Average Cost
Job	314.00	49,424.92	672.27	899.72

Figure 18: Conventional Drilling Scoreboard (1 Machine per Location)

We see that the total exits are 314 jobs in one working year with an average cost of running each job at \$899.72. The average time in the system and operation are high due to the number of machines at each location, but the purpose is to model with one machine and compare to the laser.

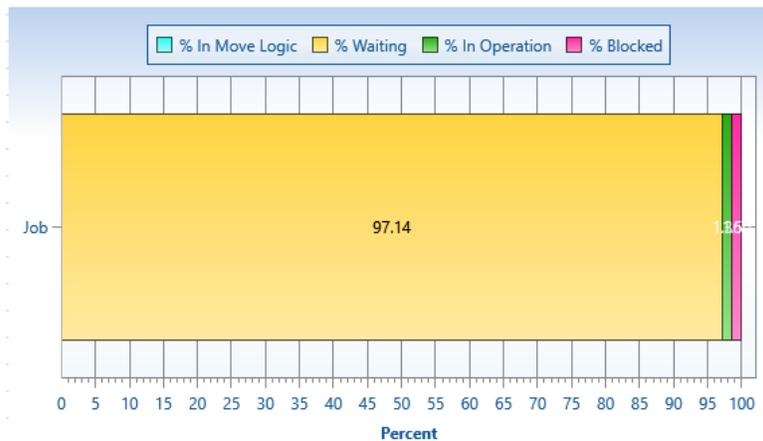


Figure 19: Conventional Drilling Entity State (1 Machine per Location)

We can see the effects in the entity state in Figure 19 where the parts are waiting 97.14% of the time. The arrival of the parts is too quick to just have one machine at one location.

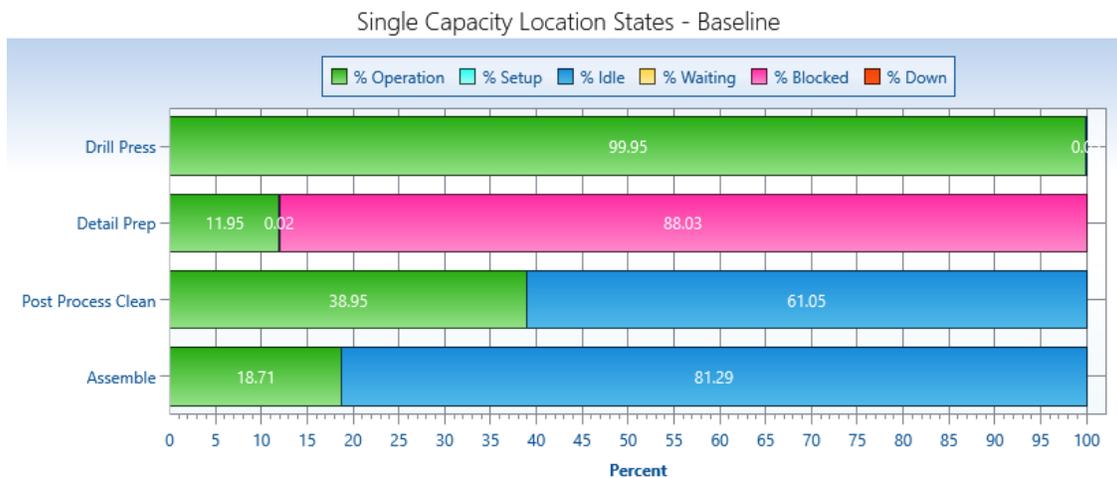


Figure 20: Conventional Drilling Single Capacity Location State (1 Machine per Location)

The reflection of capacity at each location is apparent with the one machine as well (see Figure 20). The bottleneck here is the drill press which is in operation almost 100% of the time (99.95%). With the added machines, we will see a decrease here.

Running the same simulation with the laser will allow us to analyze the differences in cost, bottlenecks, and overall efficiency. From Table 11, we can see that the laser processing is slightly faster than the conventional drilling due to the simplicity of loading parts and less down-time fixing broken drills. The post-processing is significantly less as the parts just need a quick clean and no deburring operation with multiple cleaning in between.

Scoreboard

Name	Total Exits	Average Time In System (Min)	Average Time In Operation (Min)	Average Cost
Job	352.00	47,143.01	505.37	688.84

Figure 21: Laser Drilling Scoreboard (1 Machine per Location)

Figure 21 shows the scoreboard of the one machine per location laser processing. We see an increase in total exits, a drop in the average time in the system, average time in operation and cost. This was expected due to the significant decrease in processing times.

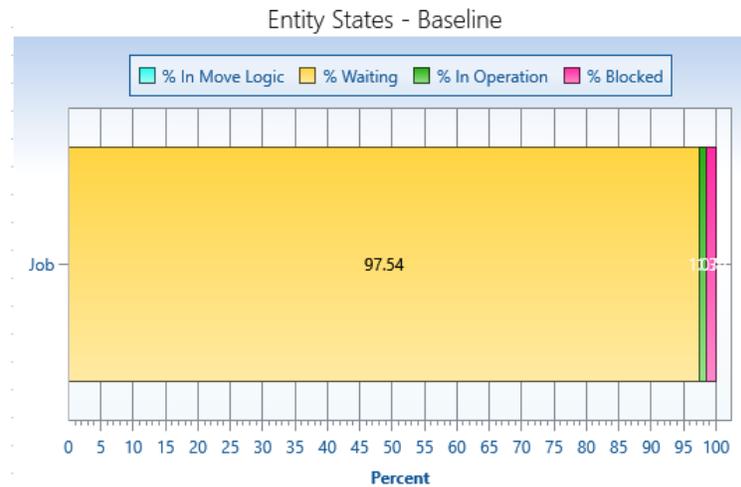


Figure 22: Laser Drilling Entity State (1 Machine per Location)

The average time in the system is spike again due to the time of the jobs waiting to be processed. If more locations existed, then the entity could be in process for a higher percentage. We see from Figure 22 that the job is waiting 97.5% of the time.

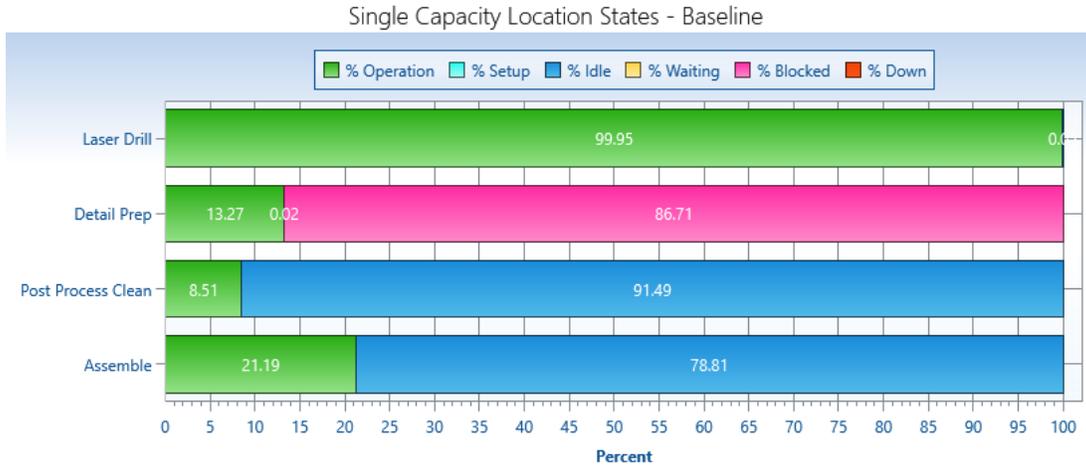


Figure 23: Laser Drilling Single Capacity Location States (1 Machine per Location)

Figure 23 gives the overview of the time in each location. Again, the bottleneck is the drilling, but we have a significant decrease of time in the post processing resulting in longer time in assembly, which leads to the increase in efficiency and job output.

It can be advantageous to view these two drilling processes side by side. The performance metrics comparison can be found in Figure 24.

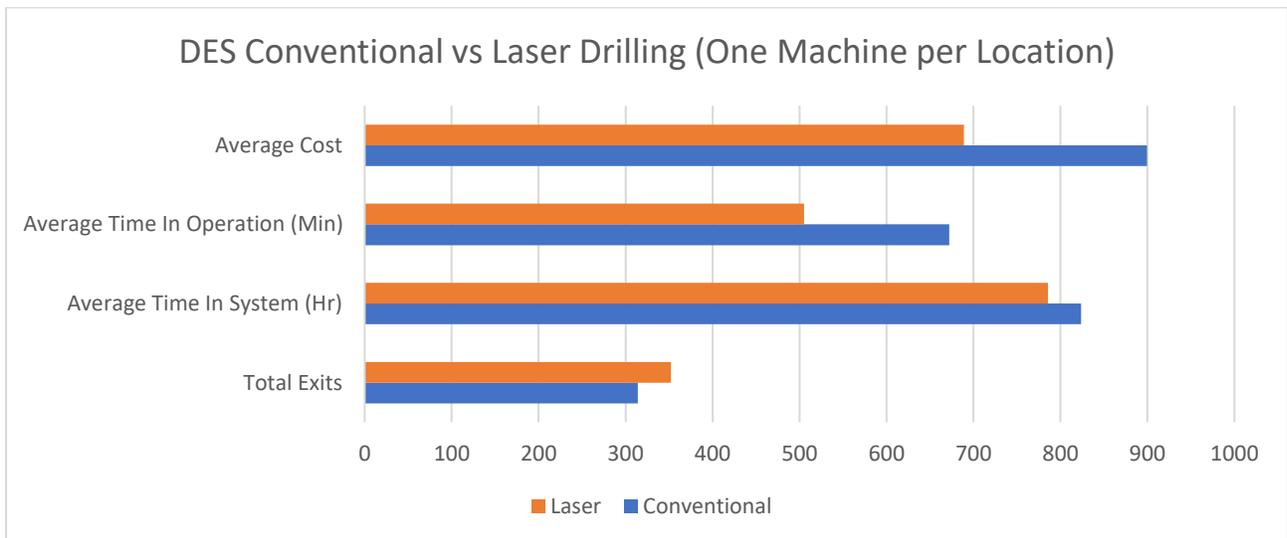


Figure 24: DES of Conventional vs Laser Drilling (One Machine per Location)

Though we see that the laser is much more advantageous, the machine itself is much more expensive than the conventional drilling machines. With less time operating and more output, it may be valuable to invest in another in the future. Before analyzing the future state situation, we must create the most realistic situation of the current state which consists of many duplications of locations and many machines for the drilling. This will most accurately depict the drilling process as it stands.

3.6 Current State Drilling Processes (Conventional and Laser) DES in Pro-Model

After looking at the processing of the conventional and laser drilling comparison with one unit at each location, we now can evaluate the current state drilling process that contains multiple units at many stations. The number of units for each process can be seen in Table 13 below.

Current State Drilling Units	
Process	Units
Arrival	1
Detail Prep	4
Conv. Drill	4
Post Processing Conv.	1
Laser Drill	1
Post Processing Laser	1
Assembly	4

Table 13: Current State Drilling Units

We build upon the conventional drilling by adding the laser drill and post processing locations and the routings for these as well where parts move to the laser if available. As conducted in Section 3.4, to update the number of units, we use the “units column” to add the units and they automatically update. Sticking with the simulation time of one working year, 2080hrs, we run the simulation.

The scoreboard can be seen in Figure 25 below.

Scoreboard

Name	Total Exits	Average Time In System (Min)	Average Time In Operation (Min)	Average Cost
Job	1,146.00	13,591.62	620.05	833.66

Figure 25: Current State Scoreboard

We see that we have produced 1,146 jobs in one working year. With some company information being proprietary, actual numbers cannot be released, but the total exits gathered is around 4% difference over the past five years. This tells me that this model is very realistic can be used for further modeling.

Figure 26 shows an increase in the time in operation, but we still have a lot of parts sitting and waiting to be processed. Figure 27 shows how often the queue is occupied at almost 96%. This is caused by the bottleneck shown in Figure 28 at the post-processing operation. There is only one station for this while we have multiple stations for drilling, so this station is being overrun. We see crowding at the laser as well, but we want this machine to be in process as much as possible.

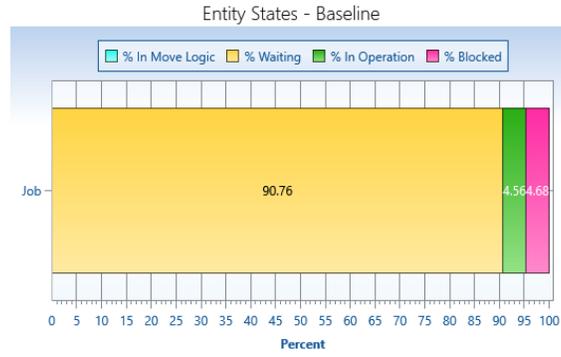


Figure 26: Current State - Entity States

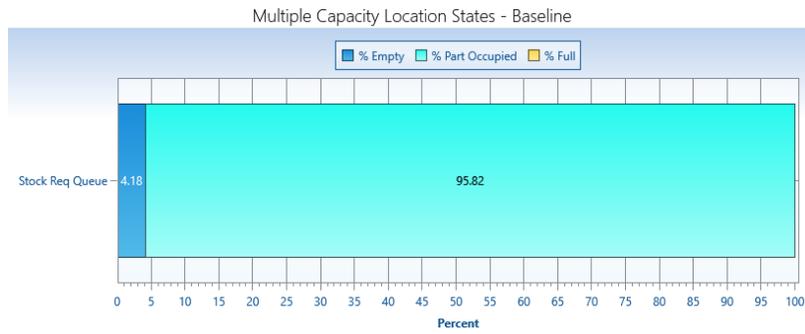


Figure 27: Current State - Multiple Capacity Location States



Figure 28: Current State - Single Capacity States

3.7 Future State Drilling Analysis

With the current state drilling simulation validated with real company data, we can proceed to try and make improvements to the process. Adding laser machines will improve overall quality of the parts and reduced the post processing time, but the laser cost is large up front. It is of interest to see if we can mitigate the bottle neck of the post-processing of the conventional drills. The cost of adding equipment to this station is significantly less than adding the laser.

We can also see that there is a lot of idle time in certain location which we can eliminate by reducing the number of units at a location or converting that unit to a new location such as an assembly location into a post-processing location. The new number of units for the future state can be seen in Table 14 and a comparison to the current state in Table 15.

Future State Drilling Units	
Process	Units
Arrival	1
Detail Prep	3
Conv. Drill	4
Post Processing Conv.	3
Laser Drill	1
Post Processing Laser	1
Assembly	2

Table 14: Future State Unit Quantity as Each Process Location

Current vs Future State Drilling Units		
Process	Current	Future
	Units	
Arrival	1	1
Detail Prep	4	3
Conv. Drill	4	4
Post Processing Conv.	1	3
Laser Drill	1	1
Post Processing Laser	1	1
Assembly	4	2

Table 15: Current vs Future State Unit Quantities

The transition of locations is simulated over one working year and the initial performance metrics can be seen in the scoreboard of Figure 29.

Scoreboard

Name	Total Exits	Average Time In System (Min)	Average Time In Operation (Min)	Average Cost
Job	1,476.00	872.86	634.93	852.56

Figure 29: Future State Performance Scoreboard

We see the total exits have increased by 330 jobs in one year and the average time in the system reduced by over 12,700 minutes with the reduction in entity time waiting and the increase of the entity in operation as seen in Figure 30. With more time spent in operation, the stock queue is empty 58.92% of the time vs a job occupying the queue 41.08% of the time which is a dramatic improvement as seen in Figure 31.

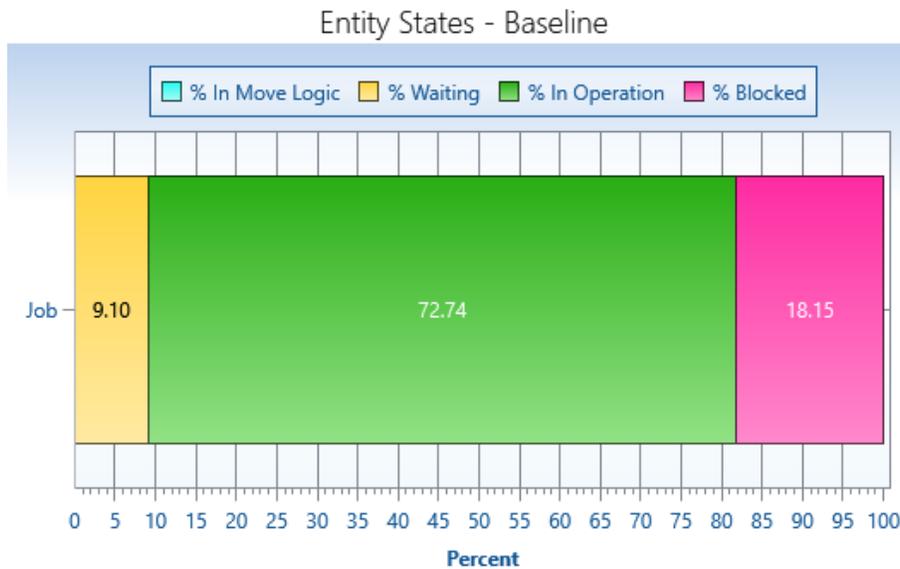


Figure 30: Future State Entity States

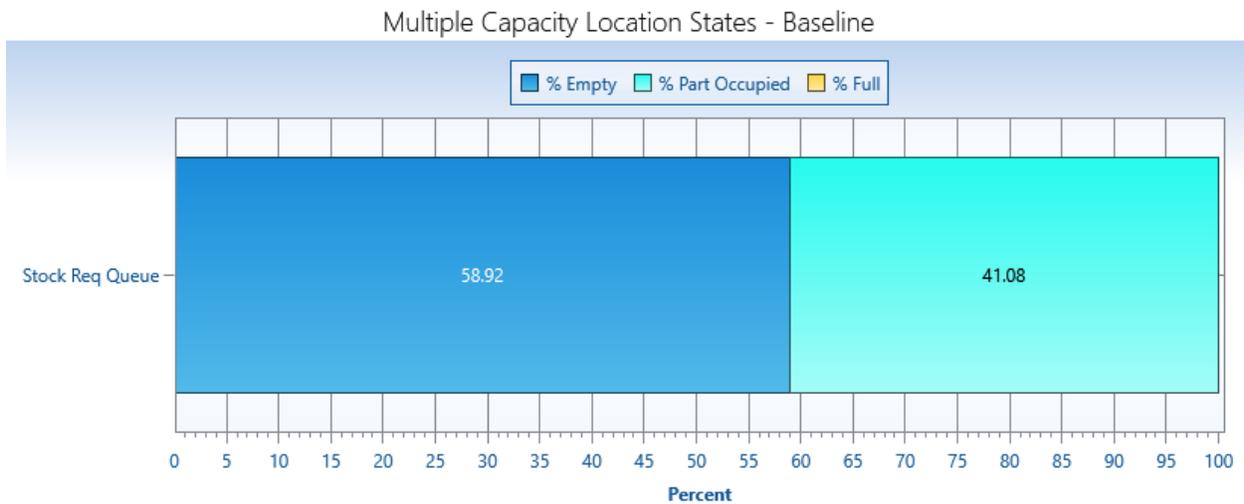


Figure 31: Future State Multiple Capacity Location States



Figure 32: Future State Single Capacity Location States

The Single capacity location states can be seen to be improved as well. We first notice much less blockage since we have more post processing stations. The assembly areas have been reduced to two locations since when occupied with four stations, two of the stations were 100% idle. The conventional drill presses also have a significant reduction in blocked states and much more time in operation.

To fully appreciate the changes from the current state to future state, we can directly compare in the same graphic. To achieve this in the best way, we filter out the representation of each unit and utilize the average of each. The comparisons can be seen below as pulled from the ProModel data.

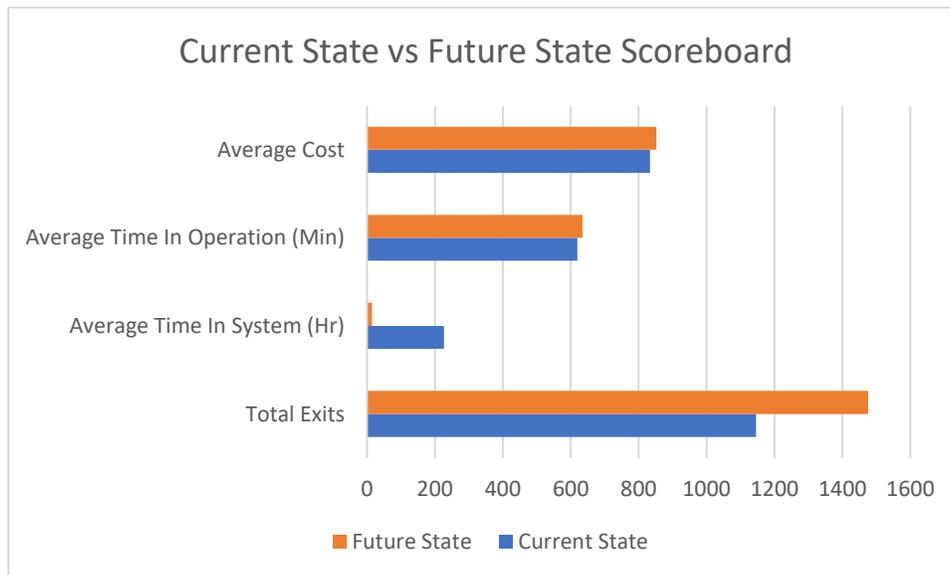


Figure 33: Current vs Future State Scoreboard

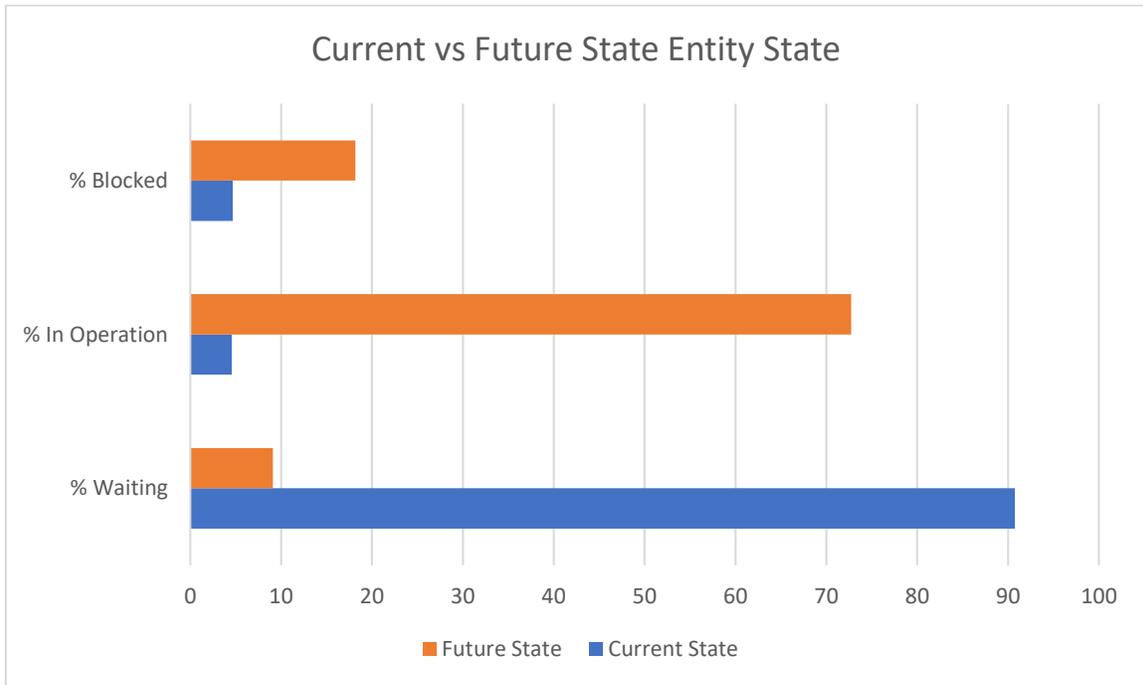


Figure 34: Current vs Future State Entity States

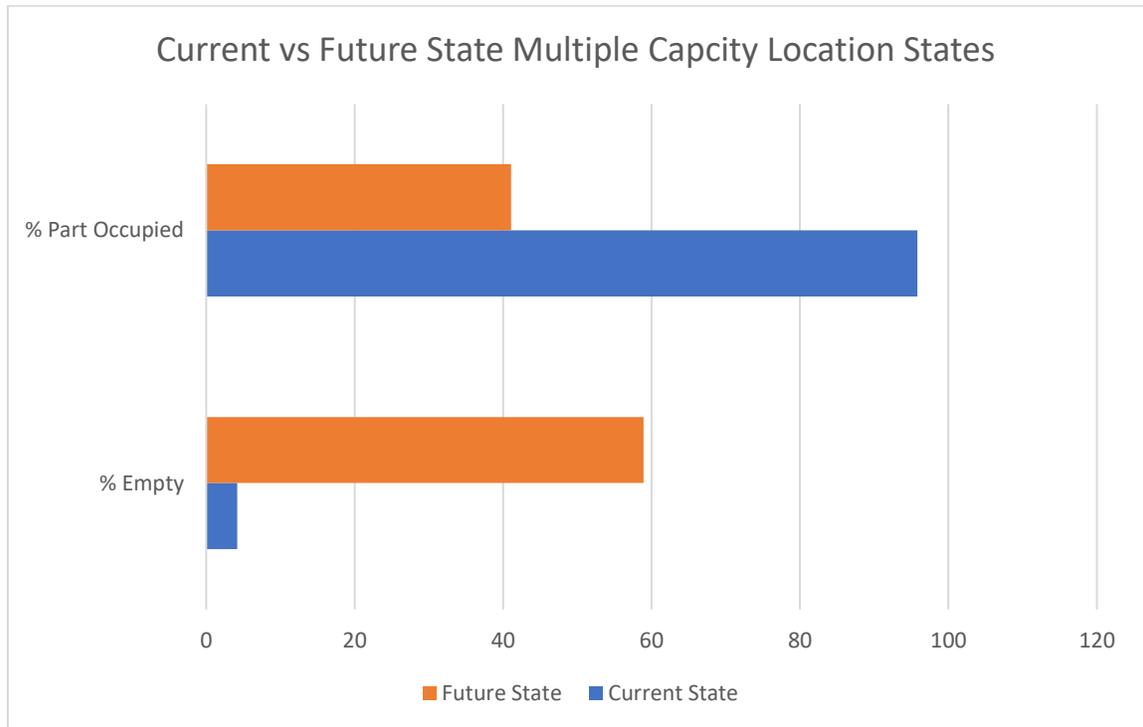


Figure 35: Current vs Future State Multiple Capacity Location States

Current vs Future State Single Capacity Location States

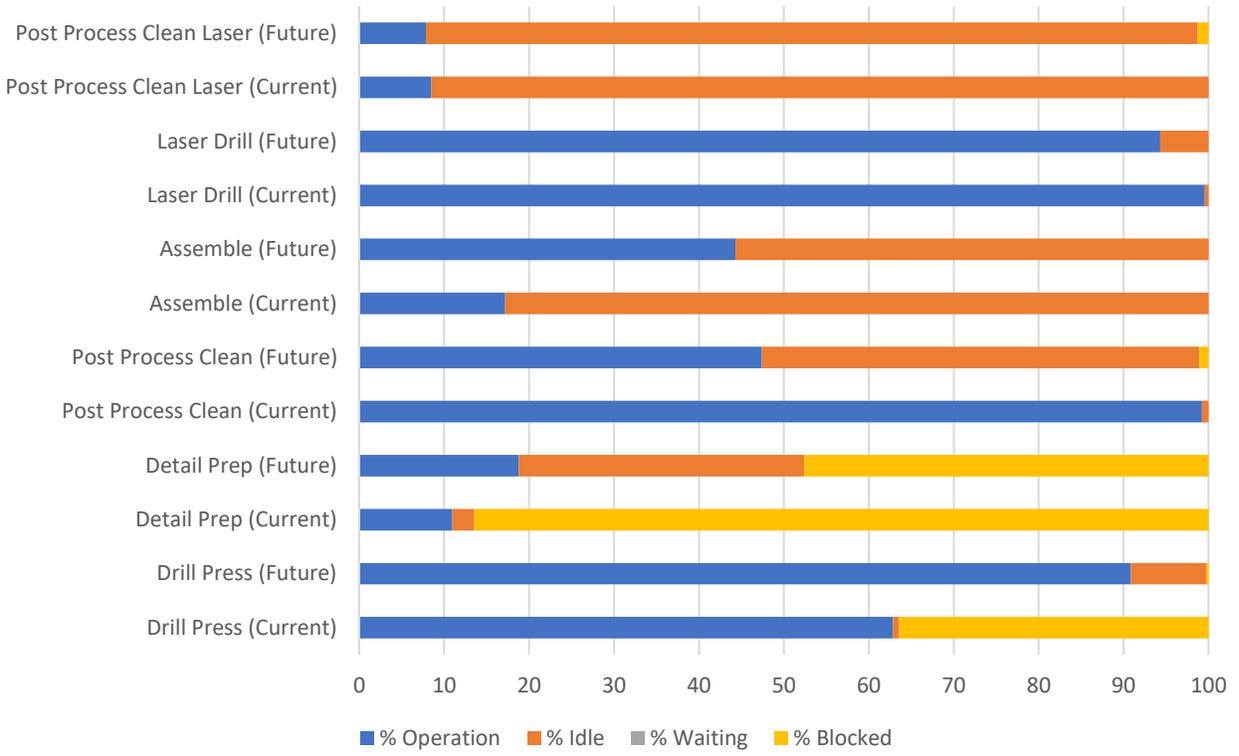


Figure 36: Current vs Future State Single Capacity Location States

4.0 Conclusion/Discussion

The steps involved in analyzing the problem state included identification of the process, narrowing down the process to focus on for the M/M/1 queueing theory, identifying the process inputs, application of M/M/1 steady state queueing theory preliminary scoping calculations, DES utilizing excel, DES convergence to steady state using excel, verification of real world expected numbers, building of a single server simulation with verification and validation, and implementation of two servers in the system for comparison.

After identifying the simplified process of the drill of the discs, I was able to find process inputs for interarrival times of the components, as well as the time to process components (when only considering one machine). This is key since the M/M/1 theory is assuming a single server queueing system. Using Little's Law and the steady state queueing calculations, I was able to calculate the server utilization and the steady state key performance indicators, all seen in Table 2. The queueing system was one of the most intriguing portions of this manufacturing logistics research. Since the DES is based on the queueing systems, it is desirable to want to understand what is going on behind the scenes of the software.

With the understanding of queueing theories and how to develop random variables for the complex manufacturing processes, I was able to create some simple DES studies in excel. The random variate portion was very interesting to uncover since I was unsure of how the randomness of a process can be modeled. There are many different distributions that describe randomness, but it is important to choose the correct distributions. Performing queueing calculations and the intricate evaluations of each event that occurs in the system shows the complexity and time that a DES software is conducting. After understanding how to calculate each step in the process and output some performance parameters, it was beneficial how to run a DES of a system that uses the same inputs as the steady state M/M/1 queueing theory. It was found with a small component and study quantity; steady state was not accomplished. This can be seen by Table 3 and 4 where Table 4 also shows the percent difference of the one study with ten components to range from 65.47% to 89.96%.

Discovering unique ways to run many studies that contains many components to evaluate in order average was insightful. The feature of the Data Table in excel allows for this, though it was not meant to be used for this function. I increased the number of components to 1,000 and still the steady state was not achieved. When running this study many times over (as the random numbers change for each study) we can average the amount of time a component spends in queue and using queueing theory equations for M/M/1, we calculate the rest of the key performance parameters. The multi-study computation is accomplished by the Data Table. After running a one-thousand component study five-hundred times, we can have a percent difference ranging from 0.32% to 1.1% as seen in Table 6. This number of calculations for excel

to perform is substantial and the setup and calculation input is very intricate but is a way to perform a DES and this is only for one small focus of the larger process. DES can easily implement these types of studies where we can define a stopping point of the program based on many parameters, such as time or a performance metric. Small changes and effects can be made very fast after running a simulation.

With the understanding of what a simulation is performing in the background, it was time to learn how to build a process in ProModel. A single server was first made where the results could be compared to the steady state calculations and excel DES model to ensure that the model was built correctly. The model was verified and validated with the server utilization showing only a 0.265% difference from the steady state calculations. Further, the key performance indicators were all below 0.4% difference with the average time in the system and queue coming in a zero percent difference showing that we achieved steady state. Using the variable and attributes in the animation allowed viewership of the time that the system achieved steady state which is ideal since we don't want to run the simulation for longer than we need to.

Before going and building a full-scale model, it is important to try and build a two-server system a few different ways and observe the results. As expected, the time in the system was reduced by half (55%) since we now have two servers, but what was not entirely expected was the 93.1% reduction in the time in the queue. If time in the queue is problematic, then it shows that adding just one machine can have a large impact, but we also suffer a bit on the idle time of the machines. Ideally, we would like them to have higher utilization though it can have an impact on longevity of the machines.

With the comfort in adding more servers, a full study can be conducted which will implement more queues and machines/stations. This way we can compare using just manual drill, laser drilling, or a combination of the two. These benefits can be seen quickly with the software and small changes can be implemented effectively to find the most optimum assembly process. With other small changes we can study the impact of longer shifts, workers, and machines and stations. Depending on the results, management can be convinced that adding more lasers or combining more laser machines with conventional drilling will lead to a more efficient and cost-effective process.

4.1 Future Work

With improvements made to the current state with the analysis and implementation of the future state, improvements can be immediately made due to the simplicity of changes. Many locations were under-utilized and with minimal cost, the stations could be transformed into post-processing stations which pose a large bottleneck in the process.

We see that the post-processing for the conventional drilling yields many problems in the process while the laser drilling post processing has a small utilization rate due to the laser drill processing quality. Future work should study the impact of introducing more laser drilling stations and phasing out the conventional drilling stations. The laser drilling machines are very expensive at around one-million dollars, so a cost analysis should be conducted to evaluate when the increase in efficiency will aid in paying for the laser equipment.

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Appendix:

Comp	RAND	Interarrival Time	Arrival Time	RAND	Process Time	Process Start Time	Process End Time	Customer Time in Queue	Customer Time in System	Server Time Idle
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.786
1	0.576	1.786	1.786	0.248	0.356	1.786	2.142	0.000	0.356	1.834
2	0.651	2.189	3.975	0.097	0.127	3.975	4.102	0.000	0.127	5.687
3	0.939	5.813	9.789	0.537	0.964	9.789	10.752	0.000	0.964	6.515
4	0.578	1.792	11.581	0.291	0.430	11.581	12.011	0.000	0.430	12.518
5	0.955	6.433	18.014	0.543	0.978	18.014	18.992	0.000	0.978	13.489
6	0.608	1.949	19.963	0.823	2.163	19.963	22.126	0.000	2.163	0.000
7	0.166	0.378	20.341	0.628	1.237	22.126	23.363	1.785	3.022	3.553
8	0.958	6.575	26.916	0.936	3.433	26.916	30.349	0.000	3.433	0.000
9	0.682	2.381	29.297	0.623	1.220	30.349	31.569	1.052	2.272	0.000
10	0.072	0.155	29.452	0.620	1.209	31.569	32.778	2.117	3.326	2.225

W _q Data Table										
2.40	1	2	3	4	5	6	7	8	9	10
1	2.34	1.51	2.11	1.72	1.57	2.47	2.01	1.58	1.86	1.64
2	2.00	1.20	1.44	1.70	1.89	1.94	2.26	2.24	2.06	1.84
3	1.85	1.99	1.57	1.40	2.22	2.00	1.53	2.63	1.76	1.65
4	2.01	2.62	1.70	1.80	2.48	1.43	1.67	1.44	1.70	2.02
5	1.52	1.65	2.19	1.79	2.23	1.66	1.78	1.44	1.92	1.62
6	1.63	1.44	1.59	2.28	1.40	1.36	1.50	1.66	2.58	1.71
7	1.50	2.04	1.66	2.18	2.07	1.56	1.83	1.70	1.30	2.45
8	1.87	2.22	2.77	1.76	1.49	2.03	1.58	1.64	2.26	2.07
9	1.72	2.17	1.68	1.51	1.95	2.17	2.07	2.32	1.59	1.82
10	2.37	1.49	1.91	1.80	1.60	1.85	2.83	1.82	1.86	1.37
11	1.99	2.08	1.81	2.05	1.73	1.30	1.94	2.00	1.88	1.49
12	2.00	1.56	1.60	2.61	2.44	1.91	1.67	1.89	1.77	1.96
13	1.74	2.39	2.04	1.62	1.71	1.31	1.27	1.56	2.27	1.83
14	1.49	1.57	2.22	1.75	2.80	1.78	1.56	2.02	1.74	2.54
15	2.21	1.80	1.55	1.49	1.53	2.29	2.15	2.39	1.76	1.69
16	1.76	1.48	2.21	1.77	2.22	1.69	1.99	1.56	1.75	2.06
17	1.85	1.64	2.55	1.56	1.85	1.87	1.82	1.67	1.62	1.73
18	2.11	1.85	2.42	1.79	1.68	1.54	2.36	1.99	1.77	1.53
19	1.37	1.76	1.53	1.87	1.61	1.88	2.28	1.83	1.91	1.75
20	1.71	2.46	2.97	1.59	1.44	1.69	2.11	2.29	1.76	2.05
21	1.56	1.68	2.05	1.29	2.28	1.56	1.80	1.70	2.50	1.52
22	1.53	1.64	1.85	1.71	1.44	2.50	1.33	1.39	1.92	1.55
23	3.06	1.83	1.68	1.75	2.75	2.08	1.67	1.25	1.86	2.45
24	1.47	1.57	1.56	1.96	2.22	2.04	1.43	1.51	1.35	1.59
25	2.19	1.94	2.08	1.77	2.28	1.75	1.80	2.06	1.71	2.37
26	1.53	1.73	1.69	2.03	1.29	2.23	1.28	2.53	1.46	1.42
27	1.58	2.76	1.73	1.56	1.19	1.88	1.54	1.72	1.94	1.78
28	1.65	1.63	1.54	1.66	1.76	1.53	1.44	2.39	1.98	1.48
29	1.93	2.48	2.17	1.65	1.83	1.94	1.57	2.02	2.44	1.98
30	1.52	2.02	2.29	1.37	2.22	1.95	2.47	1.64	2.43	2.13
31	1.26	1.57	2.76	1.74	2.07	1.67	1.94	1.68	2.09	3.00
32	2.37	1.75	1.72	1.99	1.61	1.64	1.73	2.03	1.46	1.82
33	2.37	1.89	1.56	1.98	1.65	1.63	2.04	1.66	1.82	2.02
34	1.87	2.45	3.02	1.53	2.40	1.81	1.76	2.30	1.27	1.93
35	1.93	1.89	2.01	2.54	1.33	1.82	1.74	3.39	1.35	2.54
36	1.89	1.99	1.45	1.76	1.62	2.47	2.05	1.66	1.96	2.06
37	1.74	1.77	1.73	1.94	1.83	1.85	3.01	1.87	2.10	1.78
38	1.74	2.04	1.90	1.81	2.18	1.41	1.68	2.18	1.92	1.94
39	1.36	1.04	1.24	2.21	1.66	2.54	1.72	1.86	1.78	1.63
40	2.18	1.92	1.76	1.45	1.71	1.67	1.82	1.86	2.05	1.92
41	1.82	2.00	1.69	2.92	1.73	1.78	2.58	1.66	1.66	1.78
42	2.00	1.62	2.07	1.78	1.49	1.75	2.35	1.70	2.28	3.05
43	3.57	1.70	1.93	1.90	1.69	1.46	1.89	2.21	1.21	1.60
44	2.82	1.37	2.17	2.01	2.11	2.69	1.52	1.74	2.49	1.71
45	2.92	2.43	1.42	2.38	1.93	2.09	2.16	2.05	1.84	2.14
46	1.69	2.06	2.26	1.57	2.07	2.34	2.95	1.35	2.01	2.20
47	1.34	1.30	2.26	1.67	1.87	1.29	1.54	1.91	2.02	1.67
48	1.68	1.67	2.14	1.95	1.79	1.48	1.62	2.48	1.64	1.80
49	1.73	1.13	1.41	1.42	1.83	1.56	1.87	1.74	1.77	2.06
50	1.31	2.92	1.49	1.70	1.78	1.77	2.02	1.83	1.31	1.56

